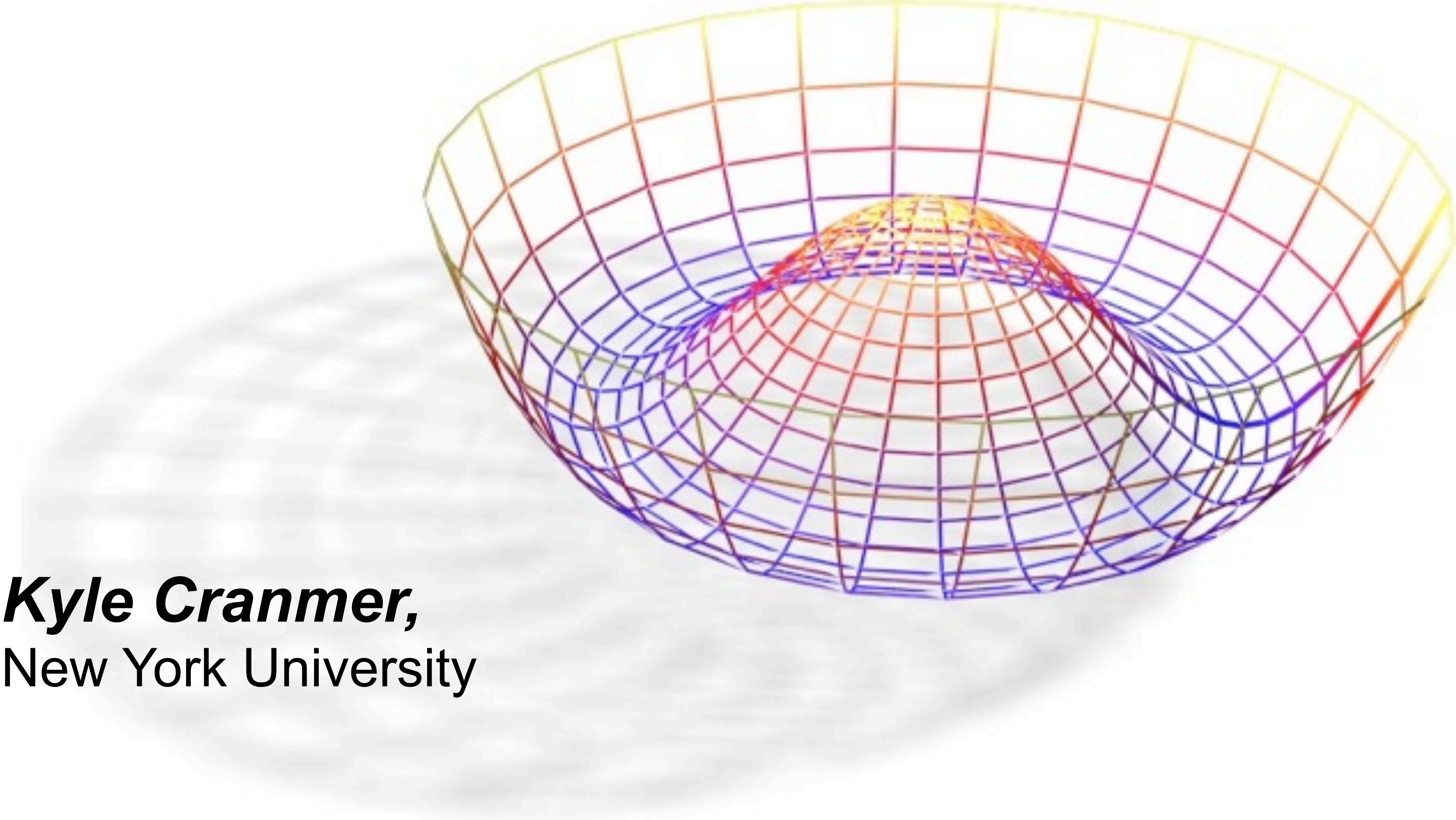
Center for
Cosmology and
Particle Physics

Recent developments and current challenges in statistics for particle physics



Kyle Cranmer,
New York University

Outline

Brief introduction to particle physics

Progress in modeling

New developments in statistical testing

Current Challenges, Open Questions, etc.

Some prospective

In just a few short years, the LHC experiments have published ***hundreds*** of papers, almost all of them making use of statistical techniques

PhyStat series and workshops have been primary venue for presenting developments and collaborating with statisticians

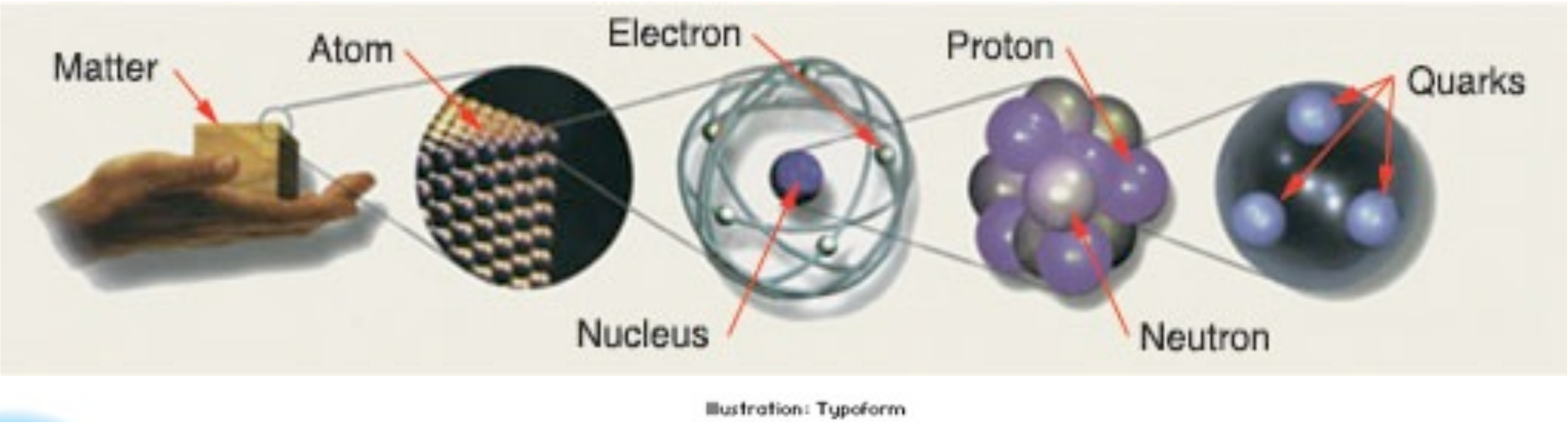
- 2011 (CERN), 2007 (CERN), 2005 (Oxford), 2003 (SLAC), 2002 (Durham), 2000 (CERN, Fermilab)
- 2010 (BIRS/Banff), 2008 (Caltech), 2006 (BIRS/Banff), 2000 (Fermilab), 2000 (CERN)

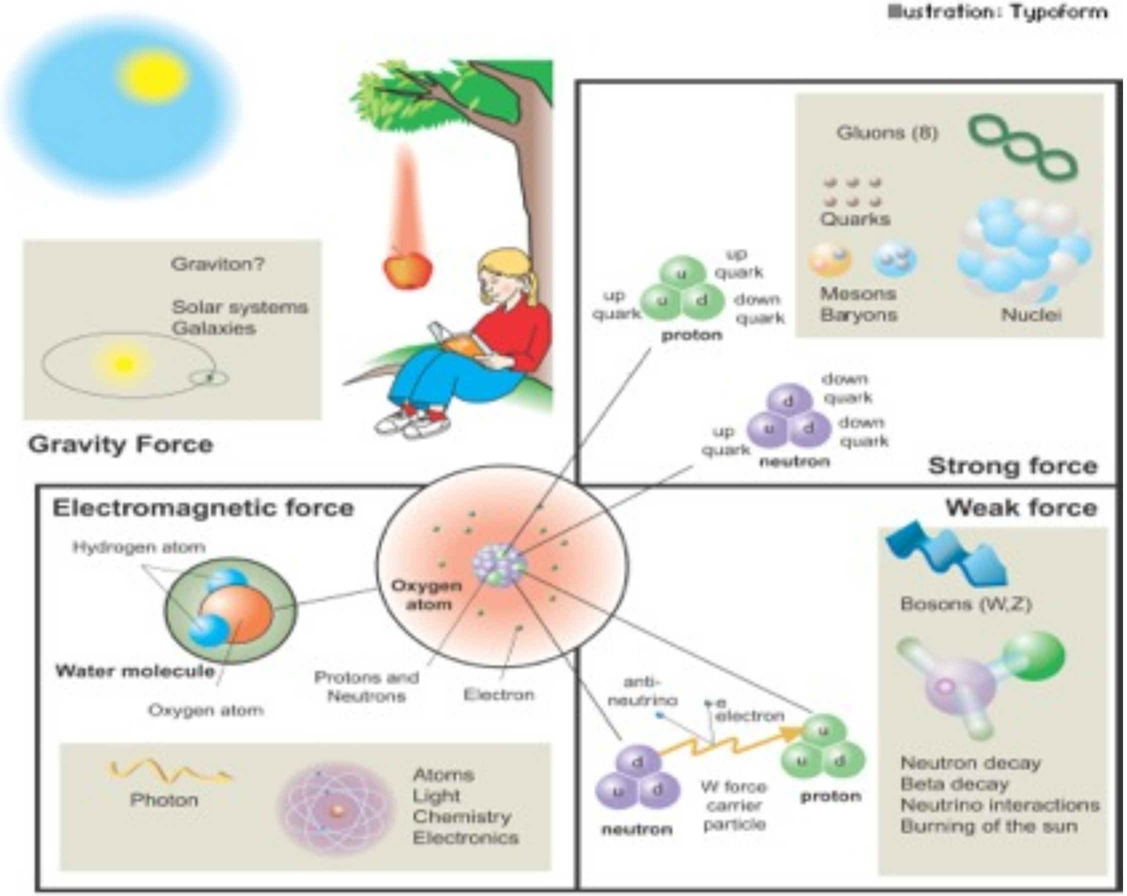
Statistical practices have evolved significantly due to this collaboration

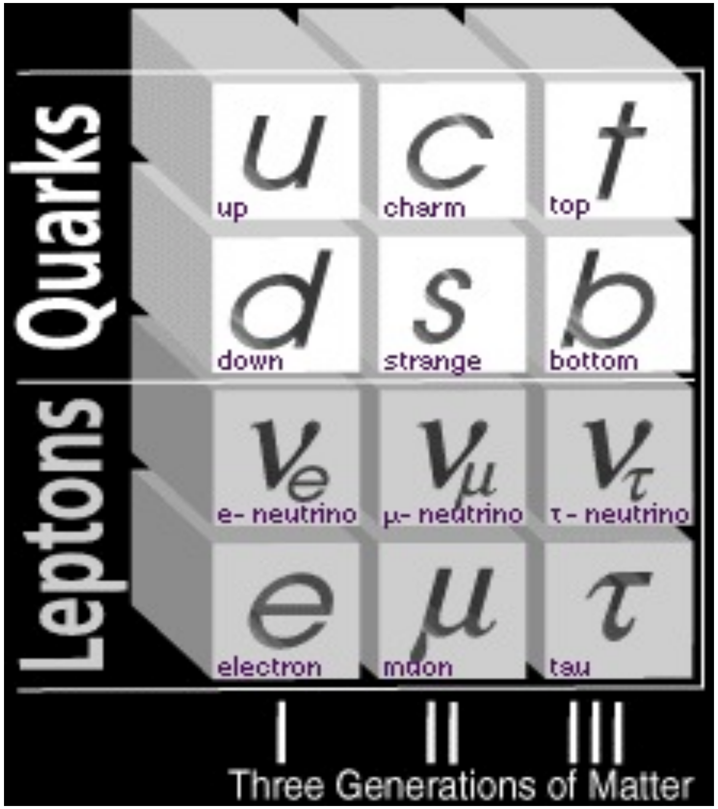
Many thanks, Louis!

Introduction

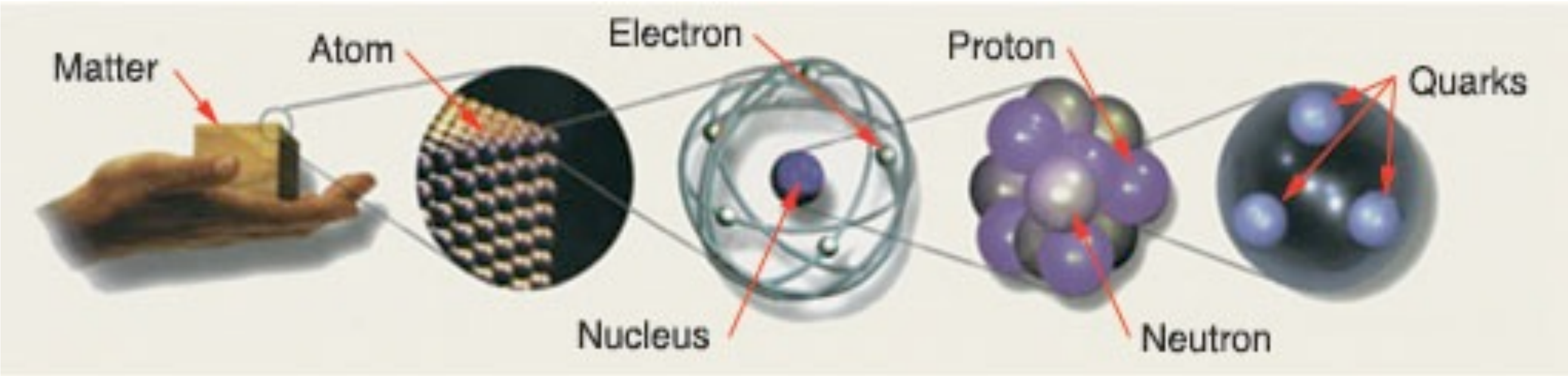
Fundamental Particles & Interactions







Fundamental Particles & Interactions



$$
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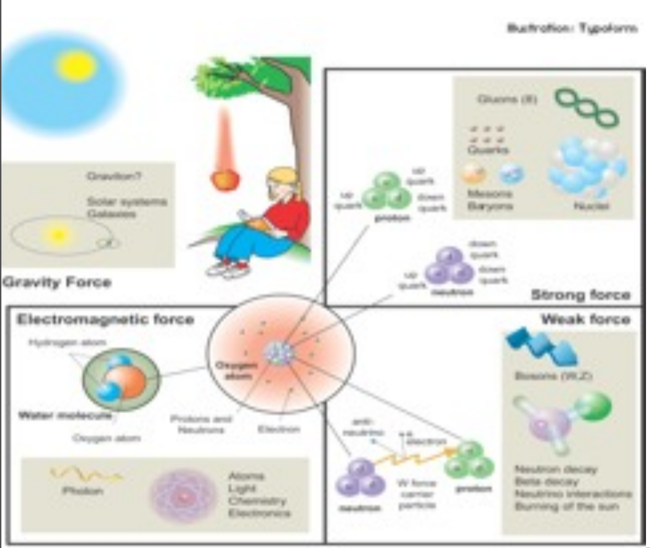

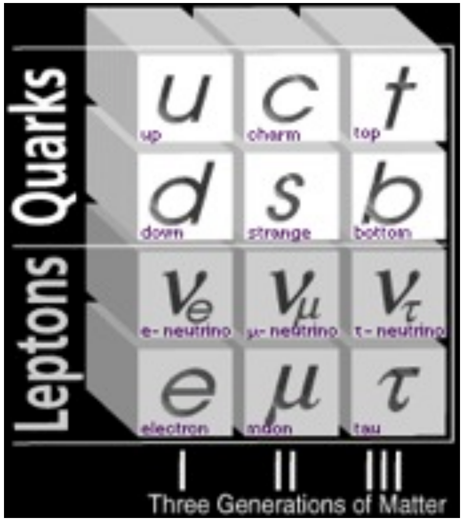

A high-level summary of the theory

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The Standard Model is encoded in a real-valued functional called a **Lagrangian** that is related to the energy of various configurations of quantum mechanical fields that permeate space-time.

- The Lagrangian is invariant to certain transformations on these fields, called local gauge symmetries: SU(3)⊗SU(2)⊗U(1)

$$\mathcal{L}_{SM} = \underbrace{\frac{1}{4}\mathbf{W}_{\mu\nu}\cdot\mathbf{W}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a}_{\text{kinetic energies and self-interactions of the gauge bosons}}$$
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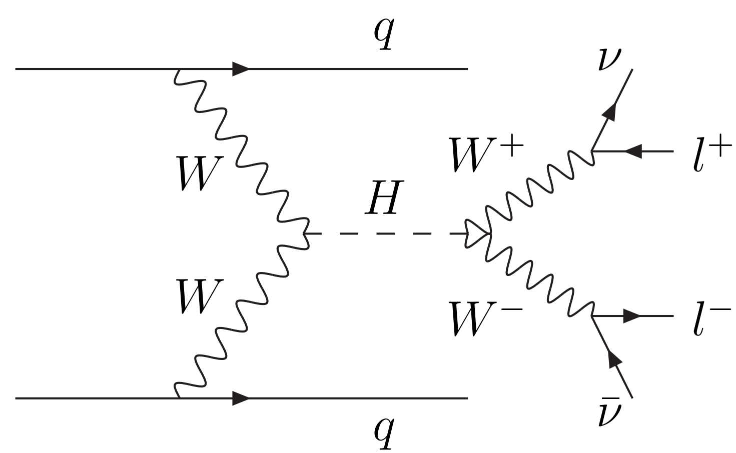
The Lagrangian is used to predict the evolution of these fields via Euler-Lagrange equations

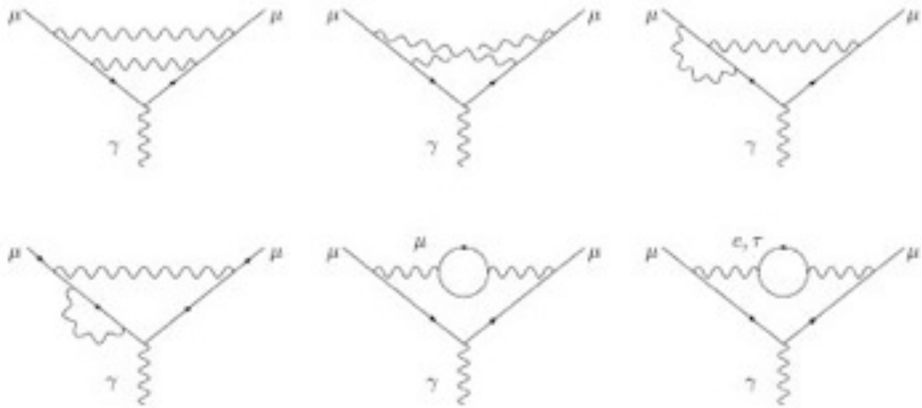
- wave-like solutions correspond to particles with a given energy and momentum
- scattering of these wave-like states correspond to particle interactions
- perturbation theory is a systematic expansion (à la Taylor series) which can be represented precisely in terms of **Feynman diagrams**
- **In some regimes** the perturbative expansion is not appropriate and **the theory is not calculable**



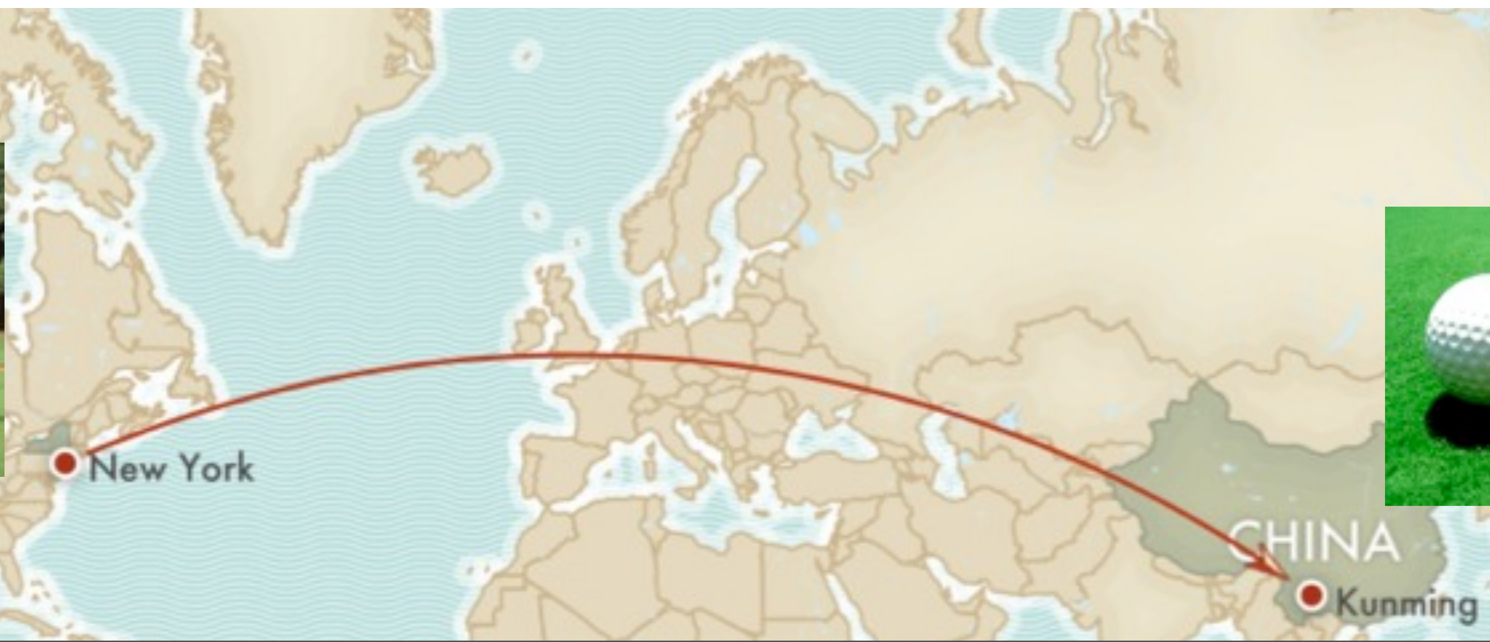
The Success of the Standard Model & QFT

Non-trivial aspects of the theory have been tested to < 1 ppm

A unique realm for reasonable statistical exploration of a scientific theory



$$a_\mu\ (exp) = 11\ 659\ 208\ (6) \times 10^{-10}\ (0.5\ ppm)$$



Energy scales in context

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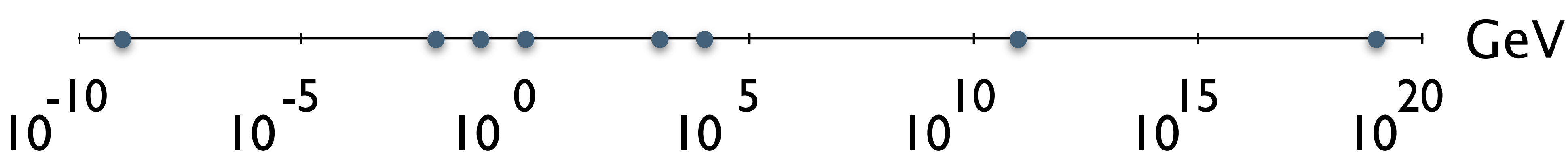

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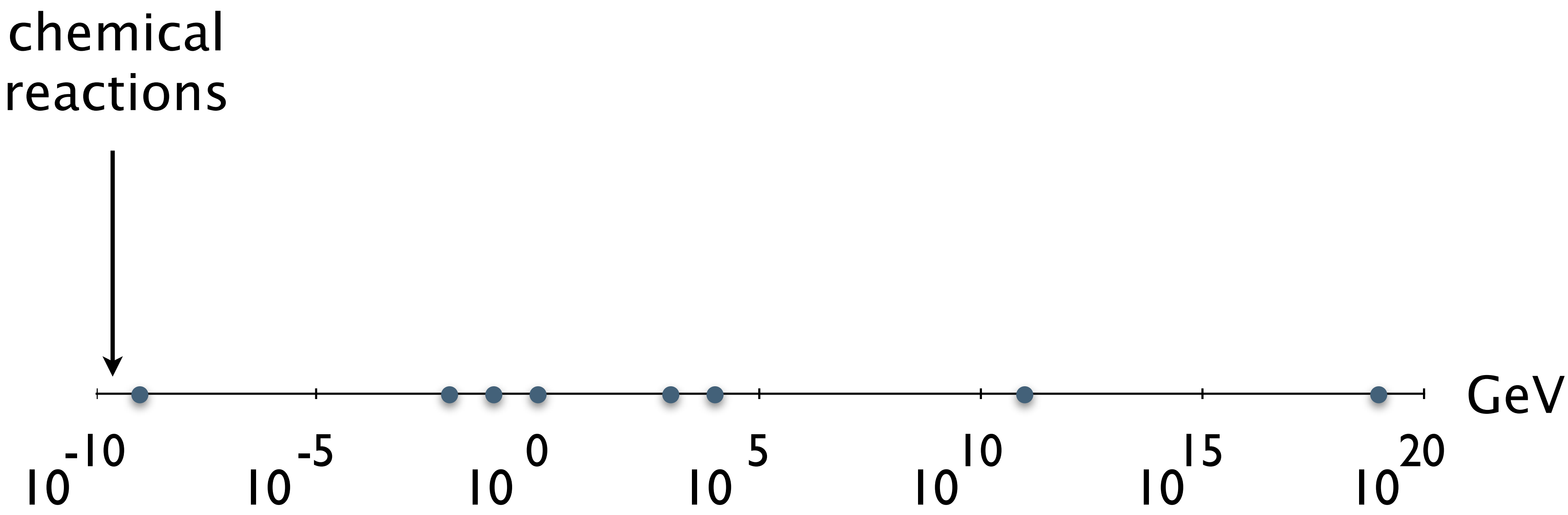

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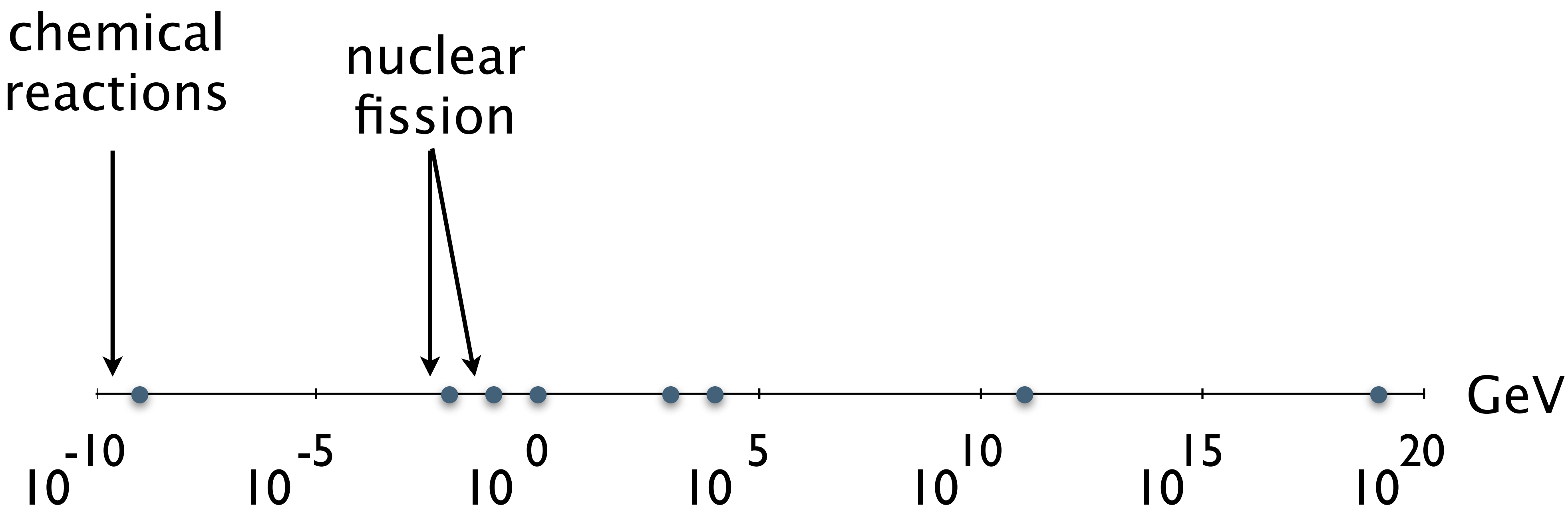

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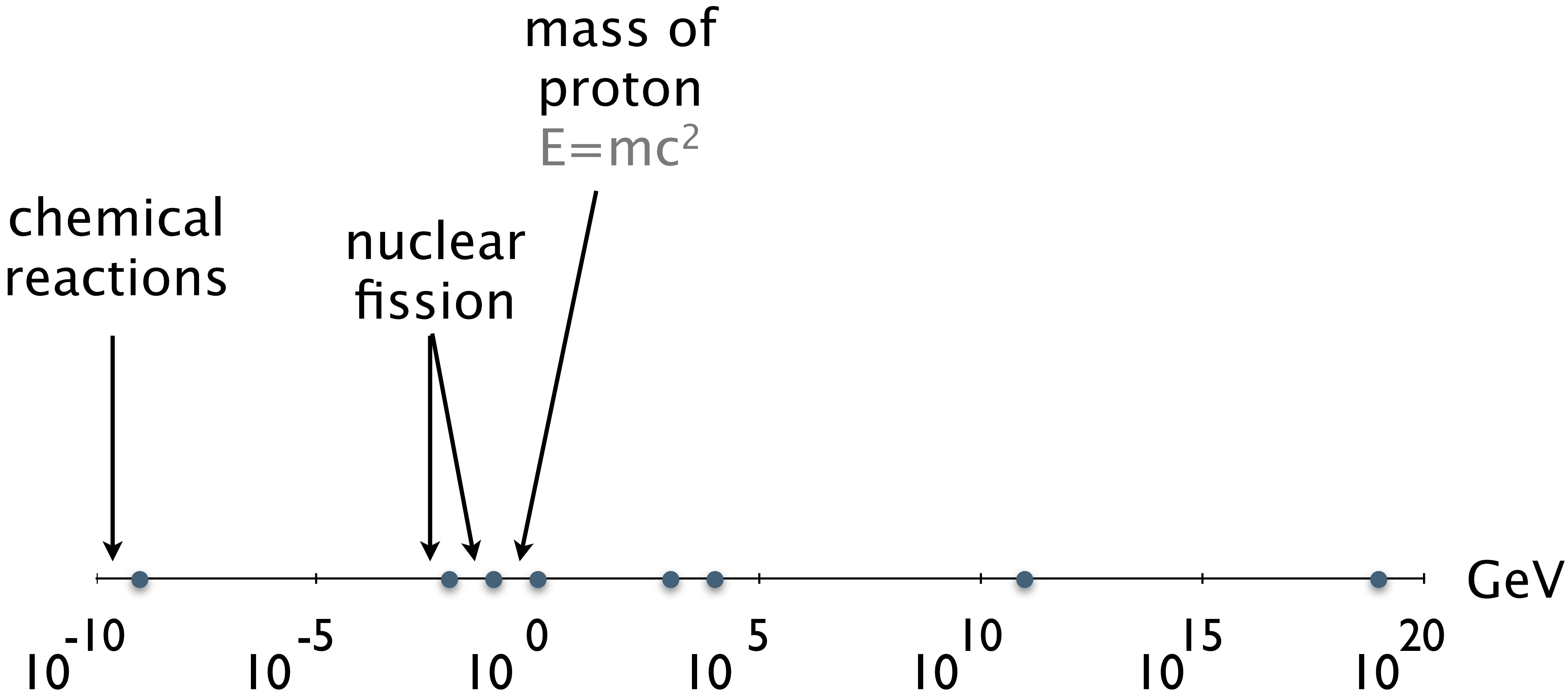

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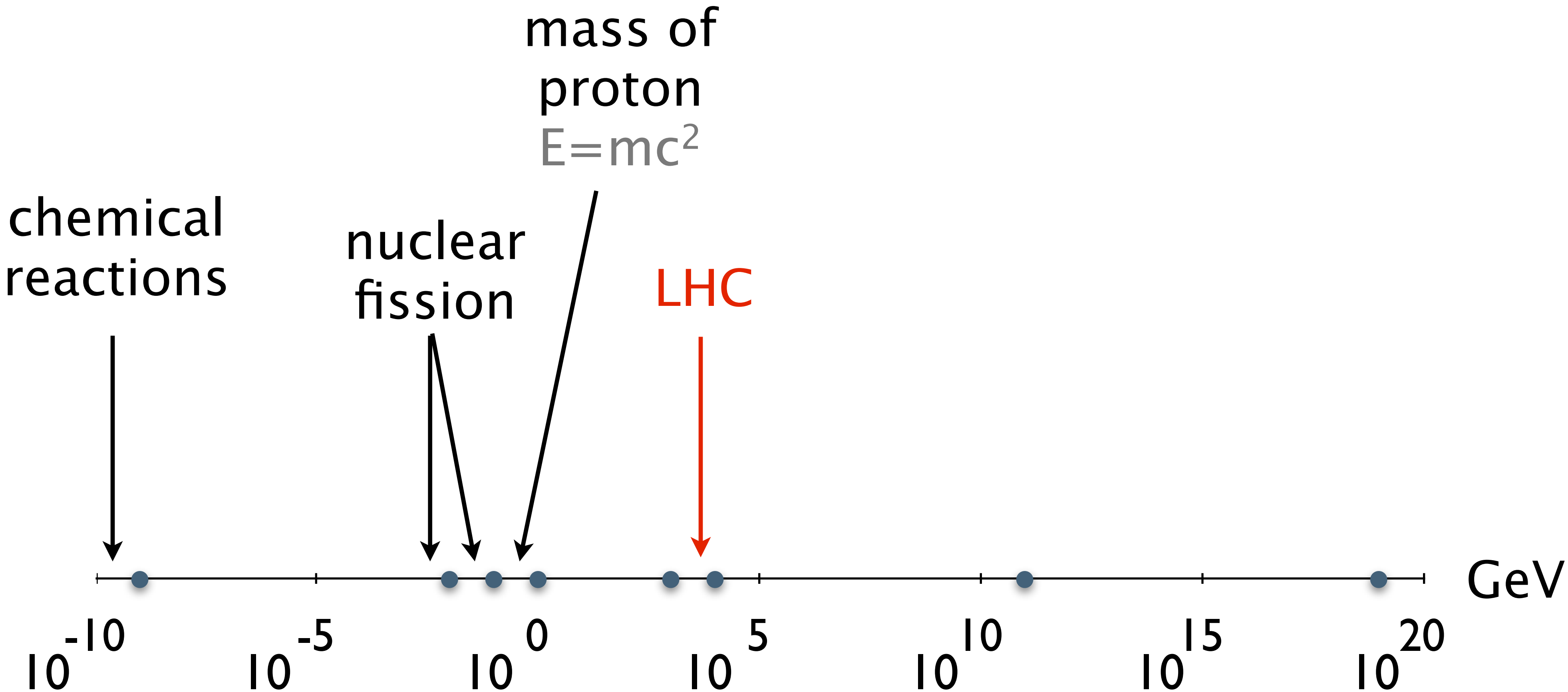

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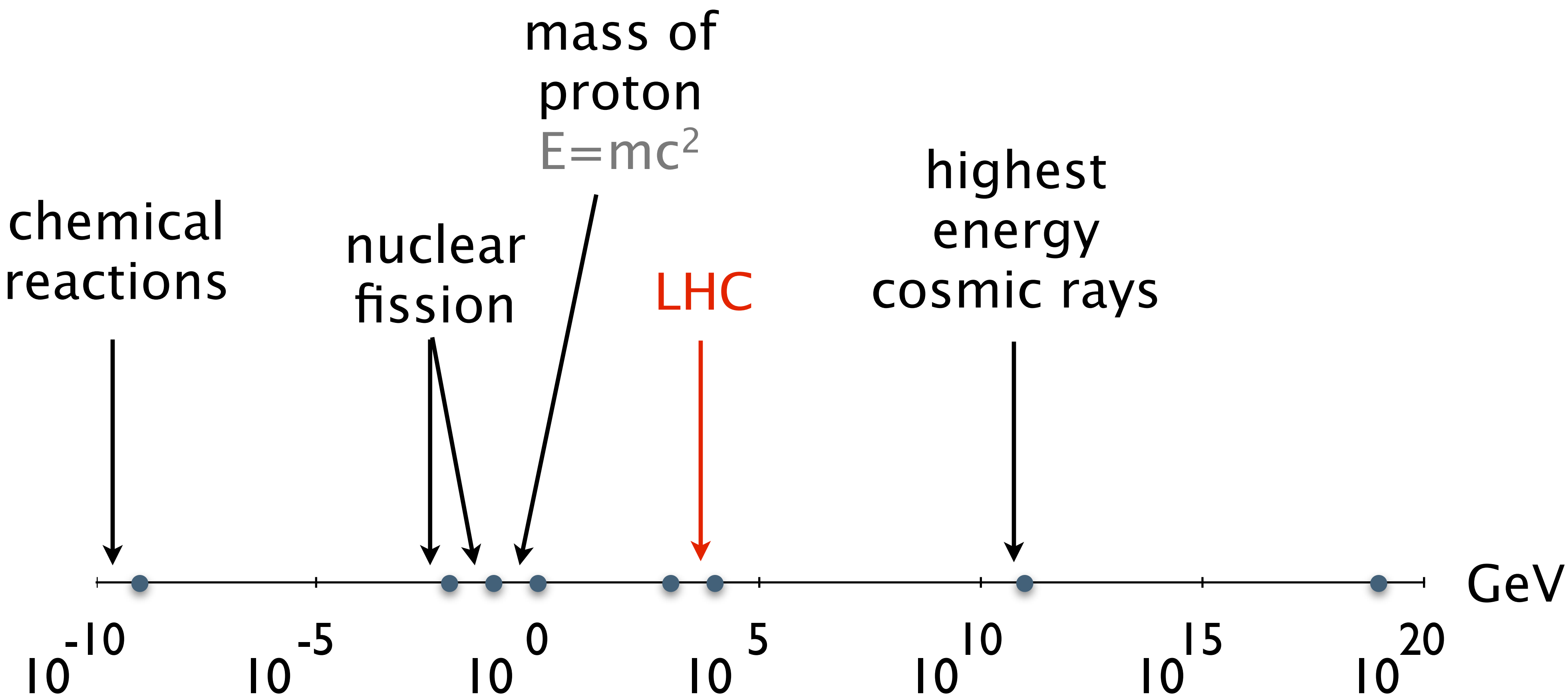

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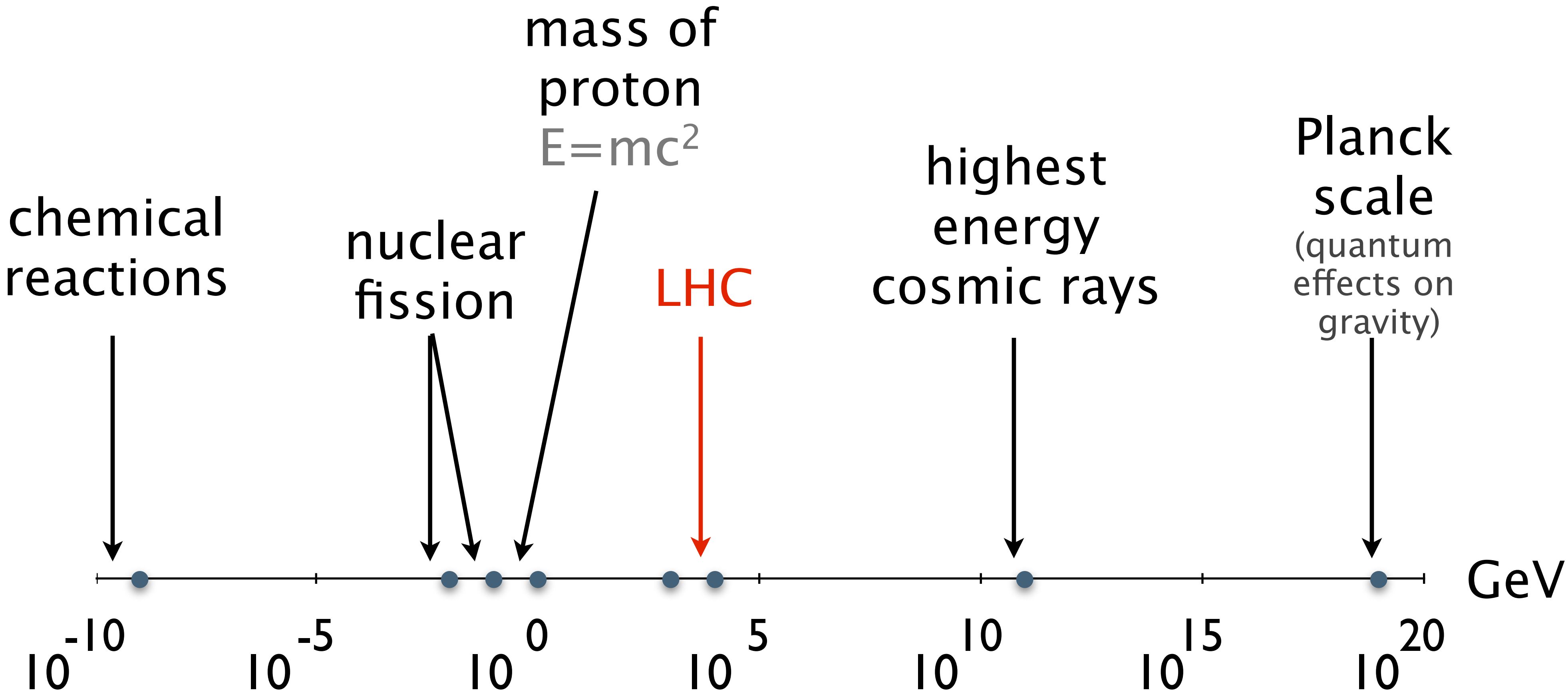

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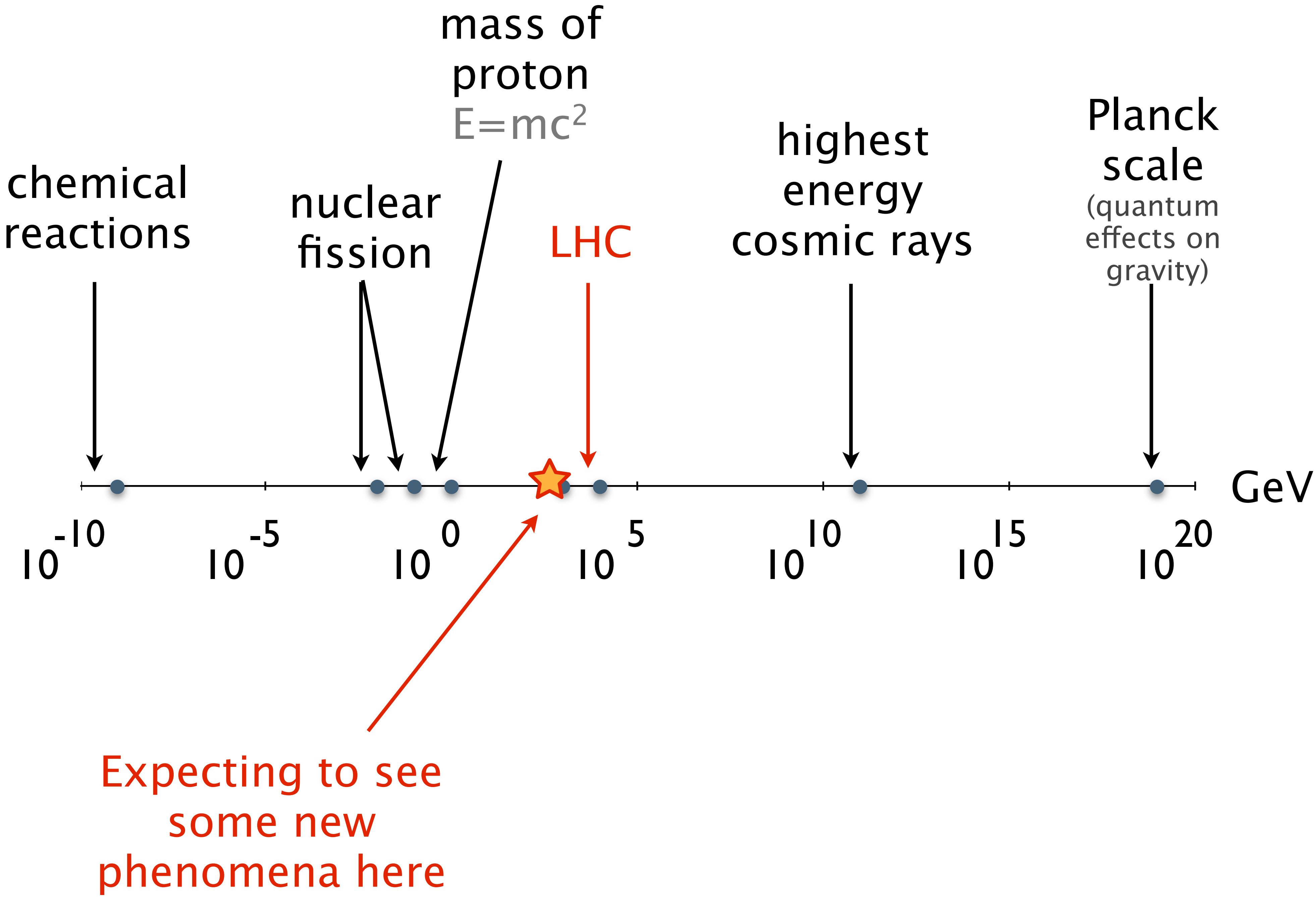

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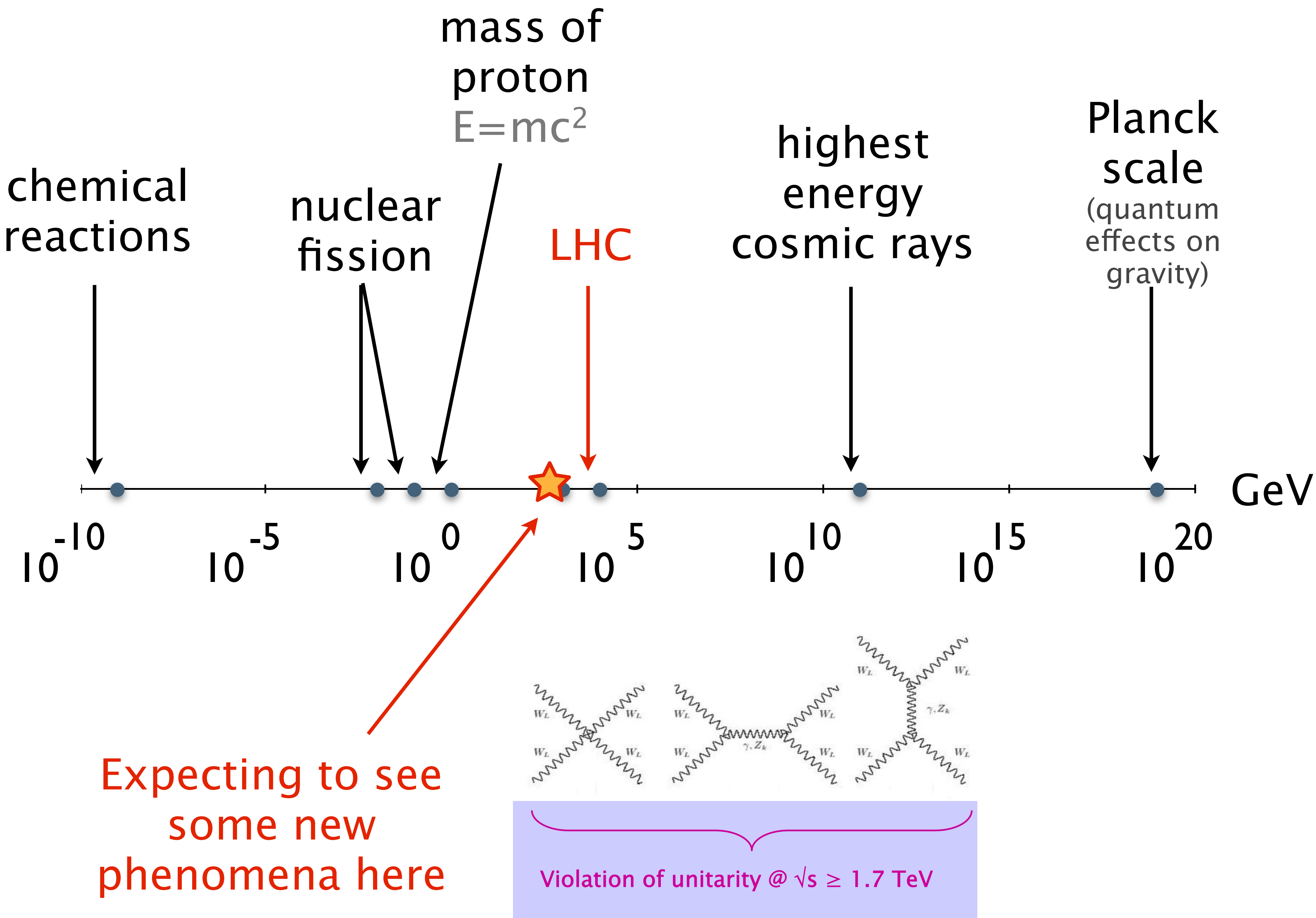

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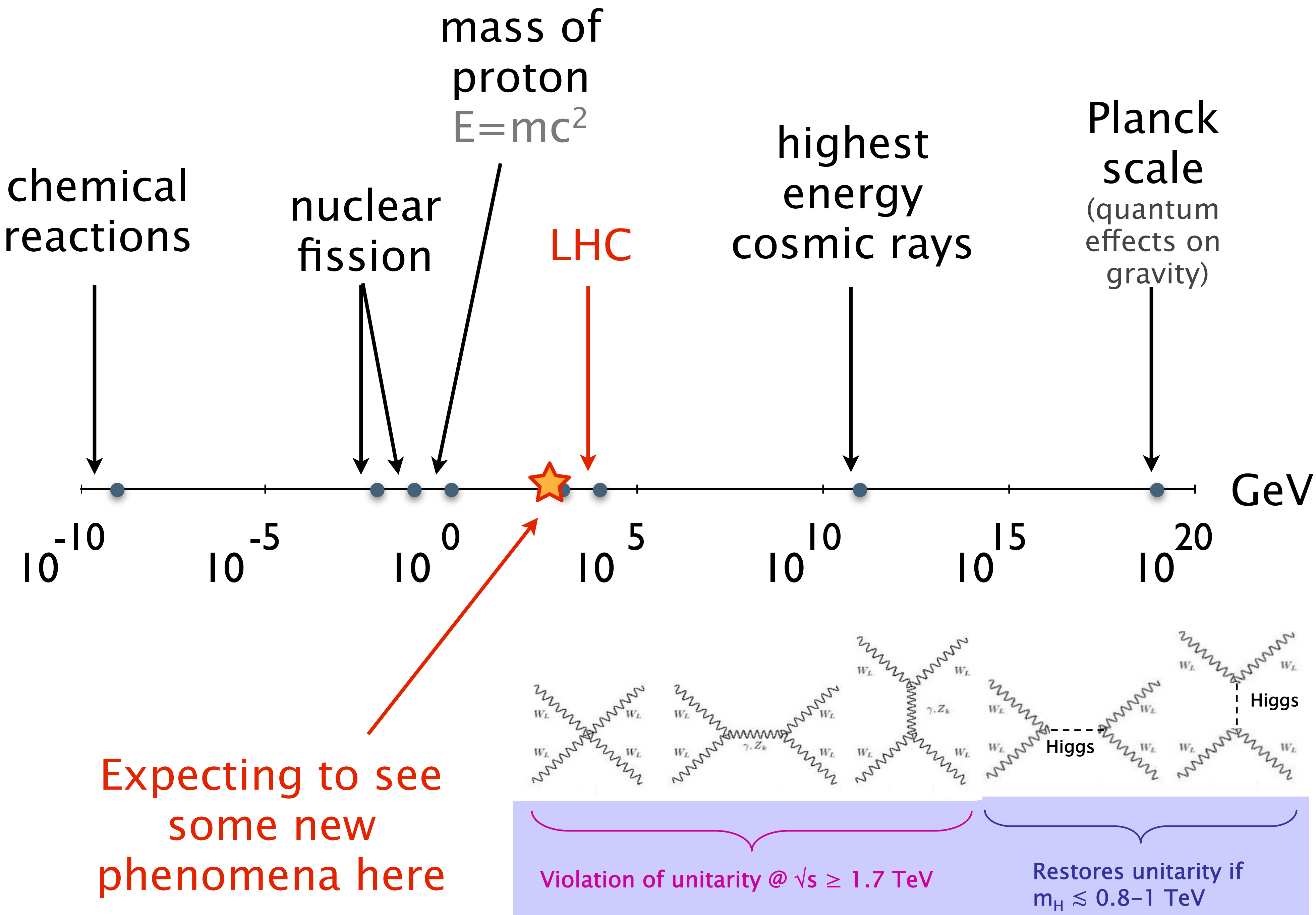

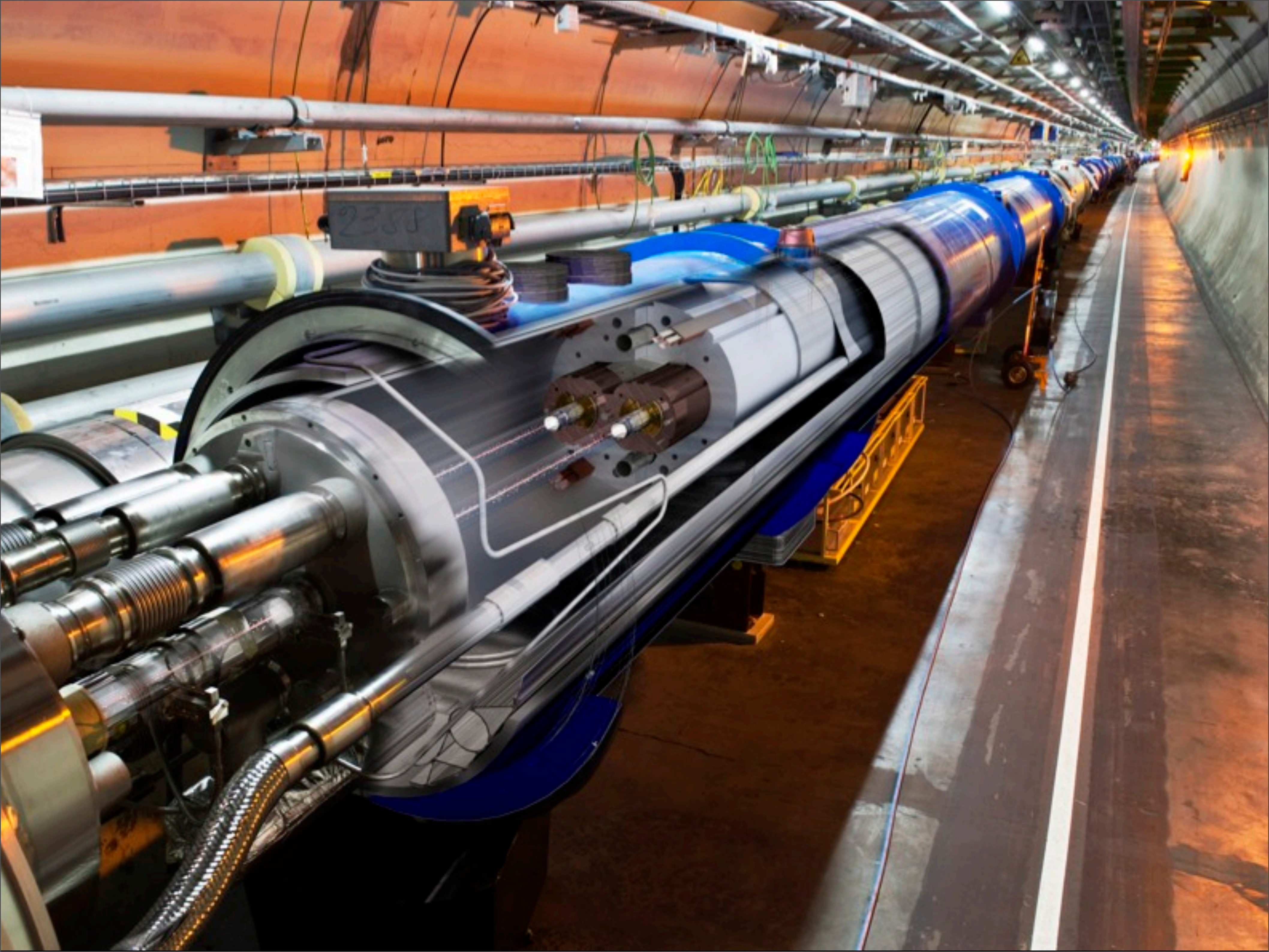

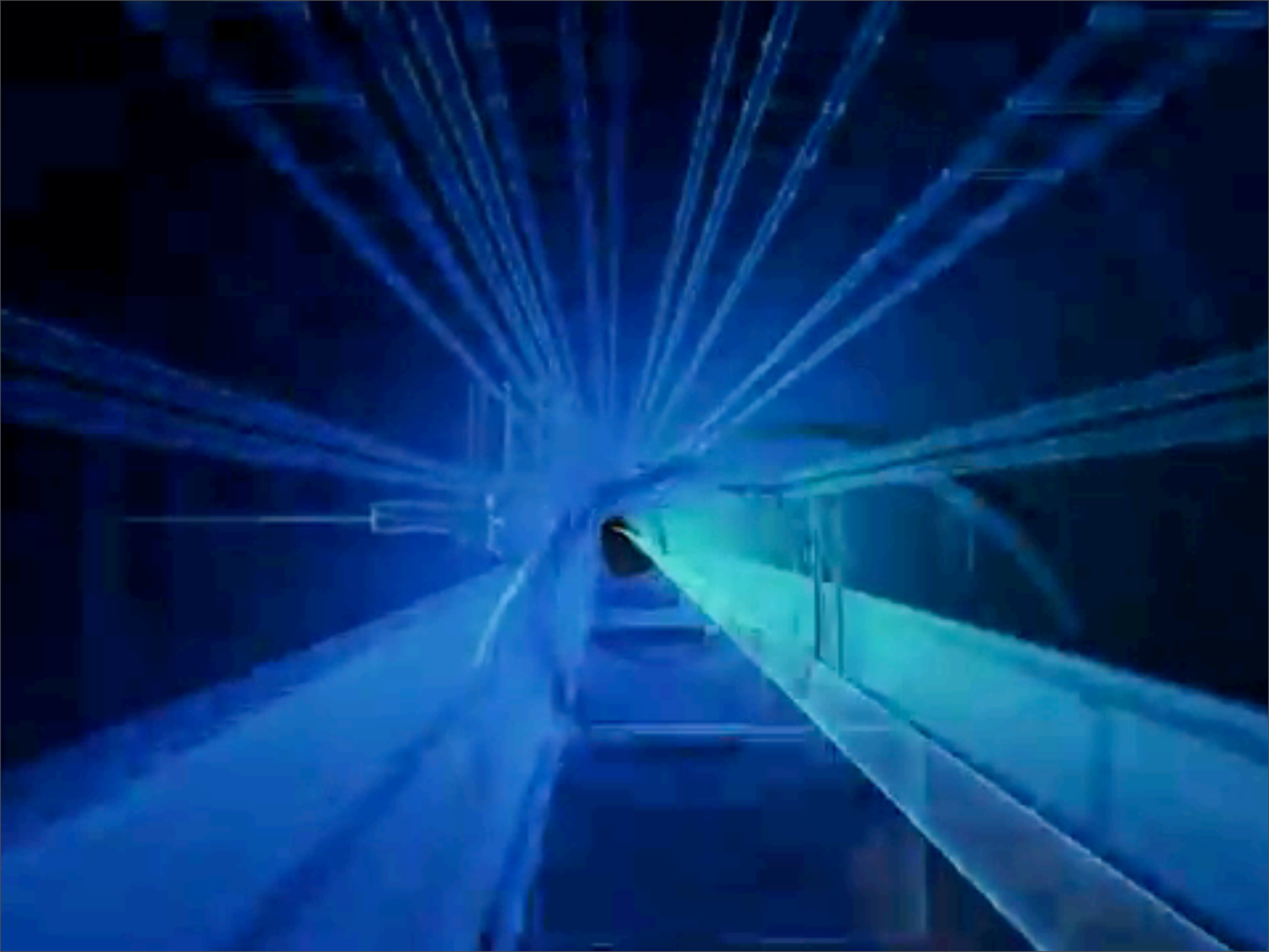

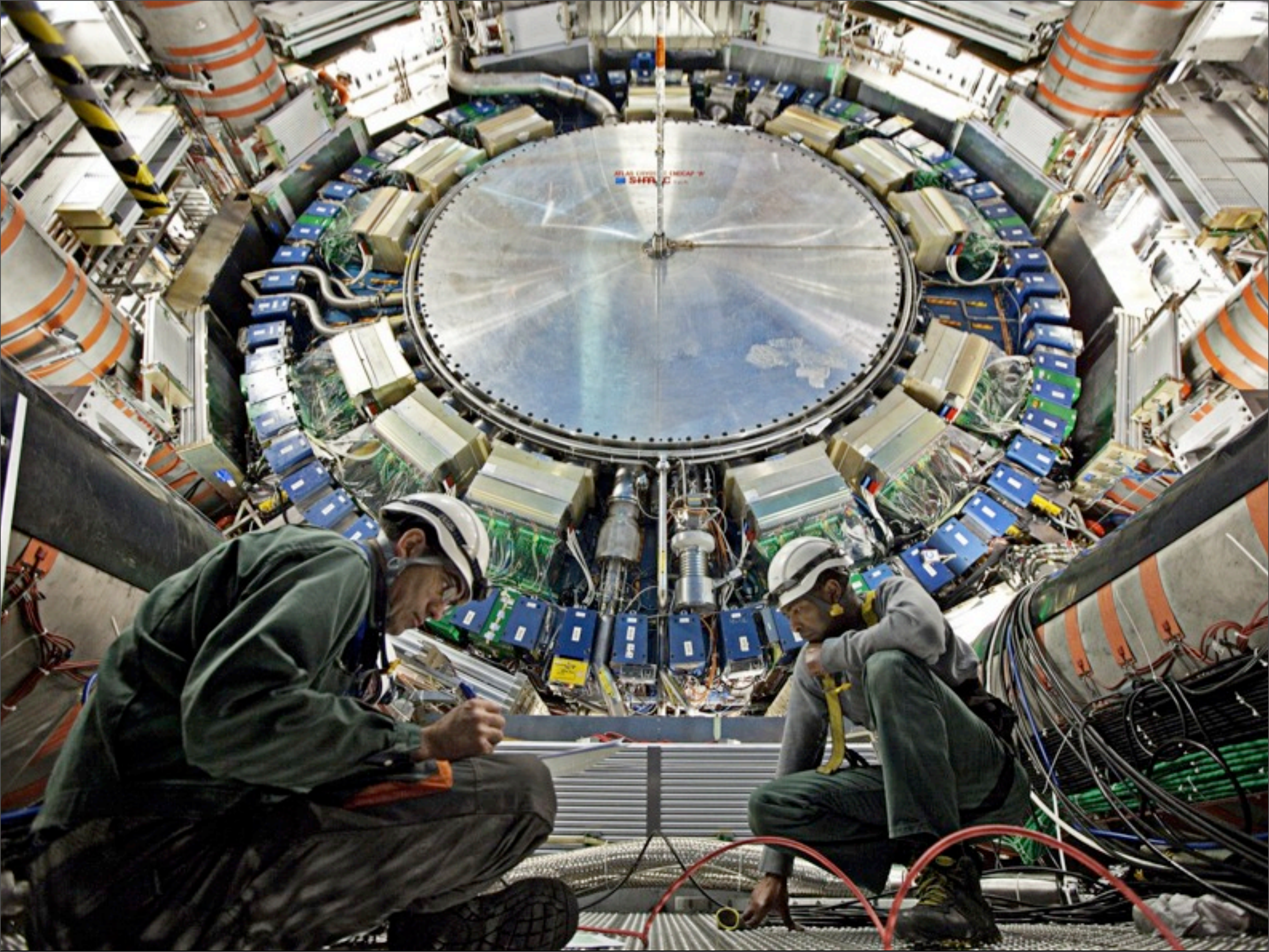

Overview of Predictions

$$\mathcal{L}_{SM} = \underbrace{\frac{1}{4}\mathbf{W}_{\mu\nu}\cdot\mathbf{W}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a}_{\text{kinetic energies and self-interactions of the gauge bosons}}$$

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1) The language of the Standard Model is Quantum Field Theory

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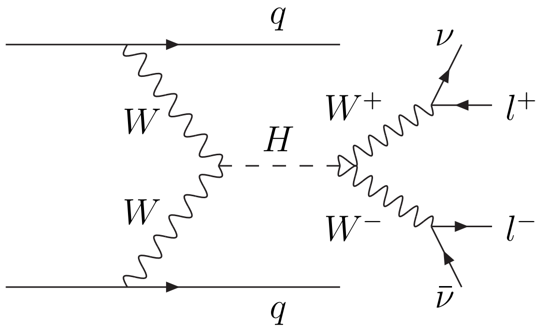
1) The language of the Standard Model is Quantum Field Theory

2) Perturbation Theory, Feynman Diagrams, and Factorization are used to construct Monte Carlo simulations of the interactions



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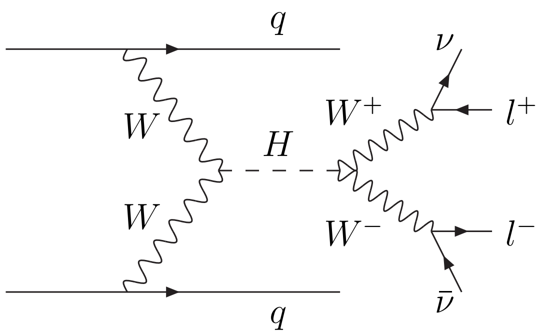

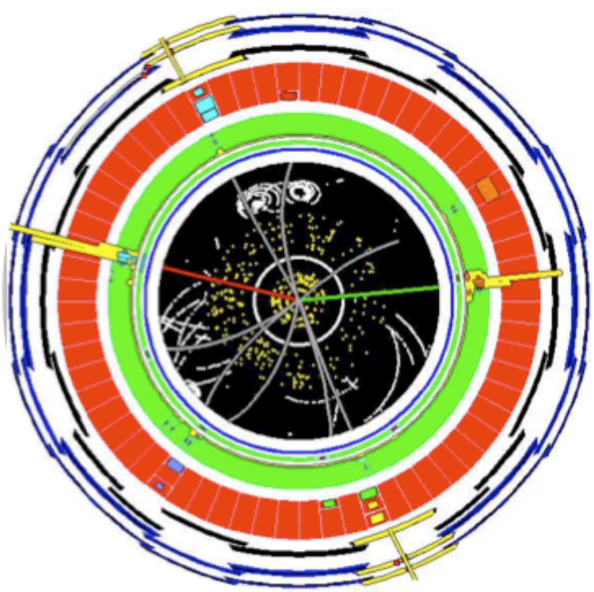

3) The interaction of outgoing particles with the detector is simulated.

Overview of Predictions

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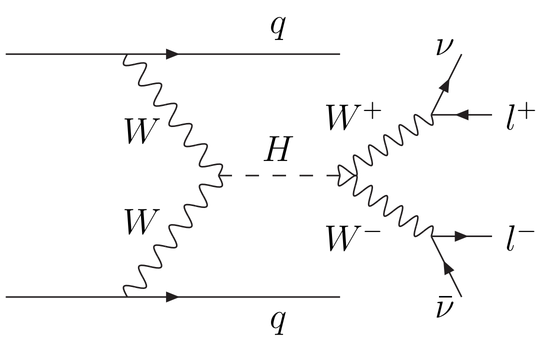
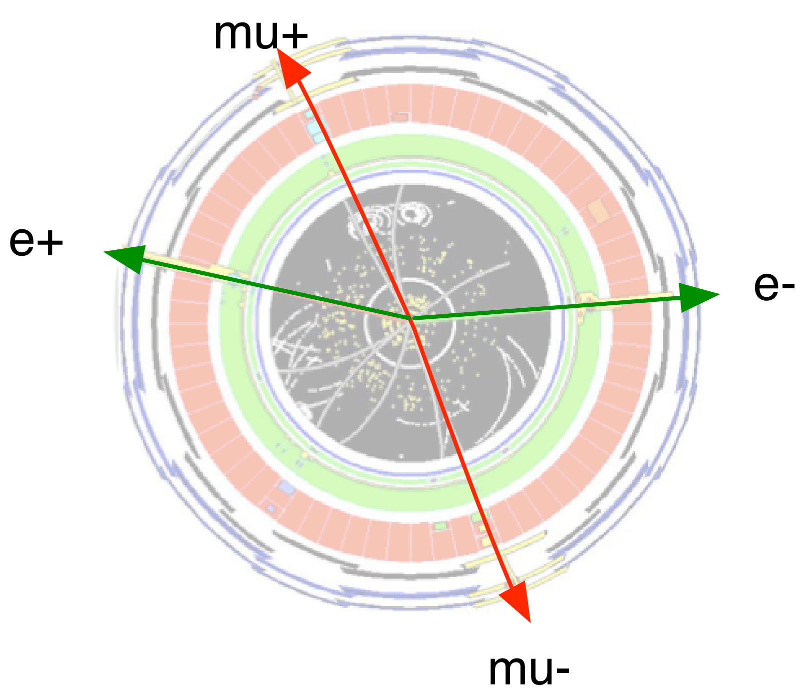




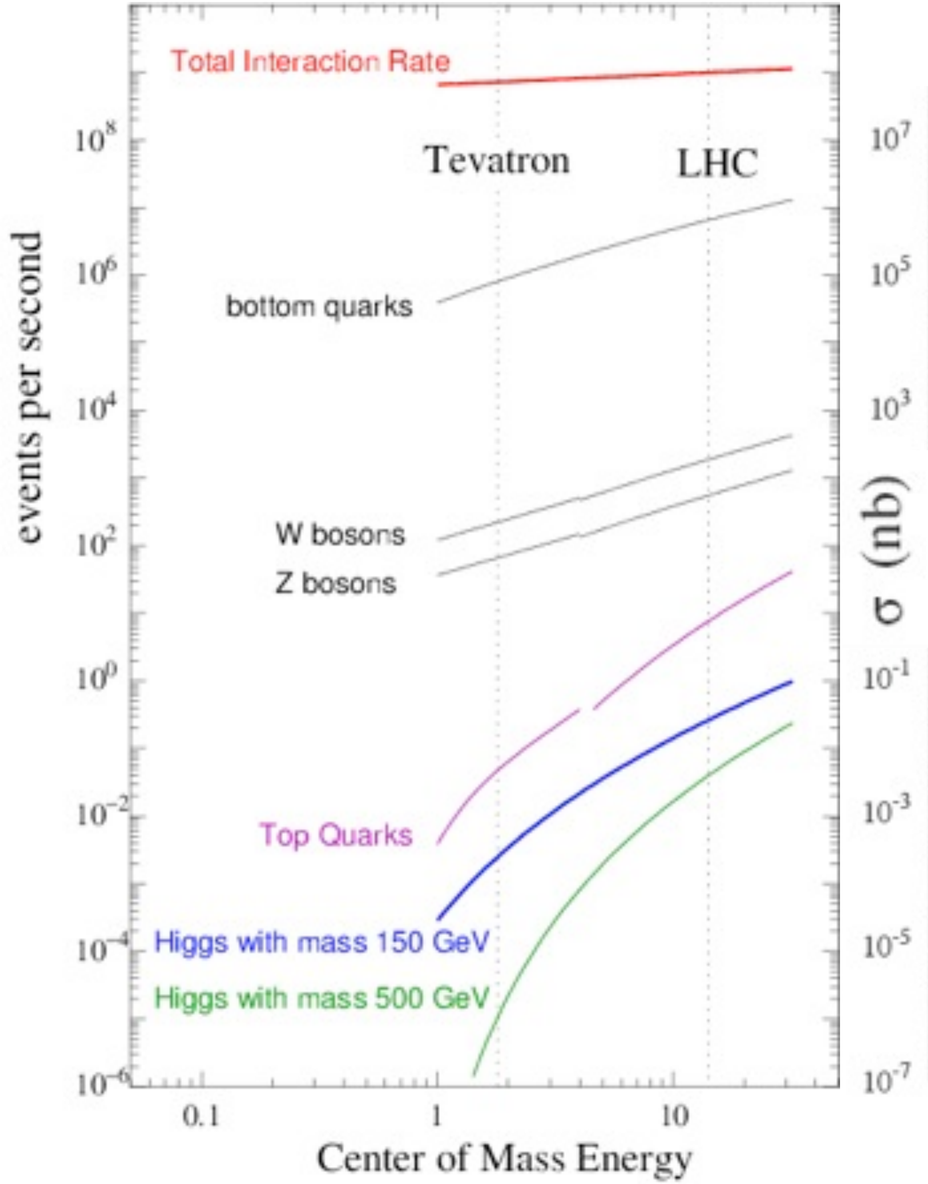
3) The interaction of outgoing particles with the detector is simulated.

4) Finally, we run algorithms on the simulated data as if they were from real collisions.

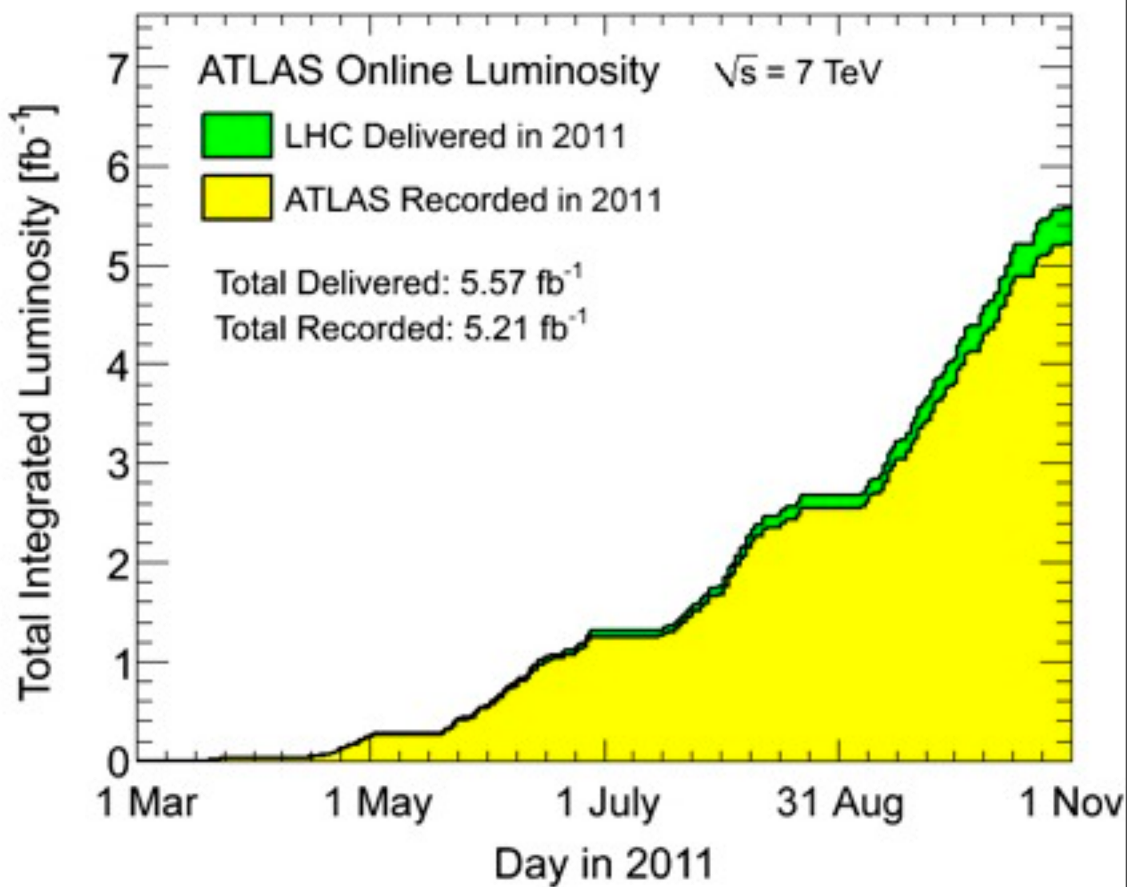
Number of collisions in 2011

expected number of scatterings = cross section [cm^2] x Luminosity [1/cm^2]

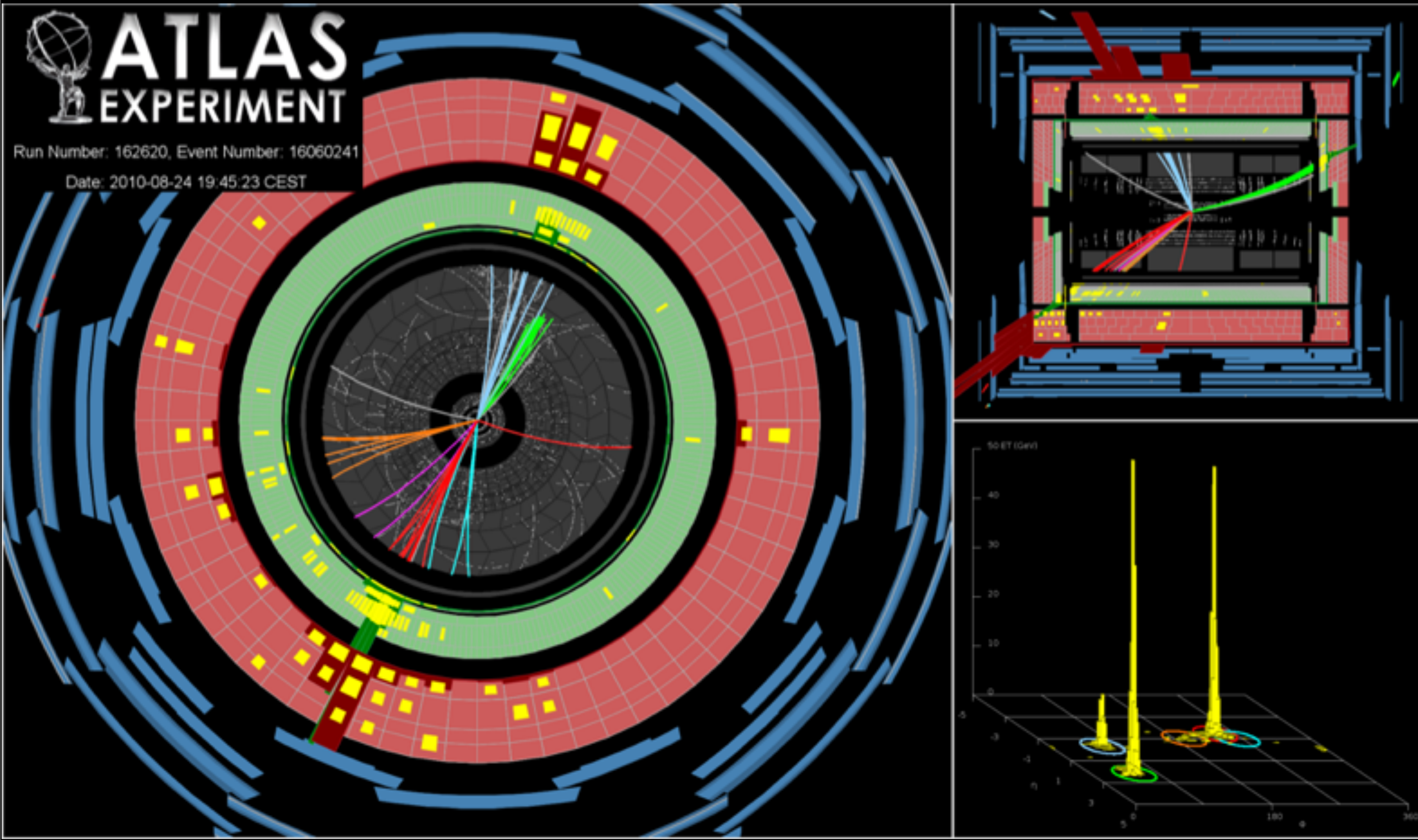
$$\langle N \rangle = \sigma L$$



$80 \text{ mb} \cdot 5 \text{ fb}^{-1} = 4 \cdot 10^{14}$ collisions



$1 \text{ nb} = 10^{-33} \text{ cm}^2$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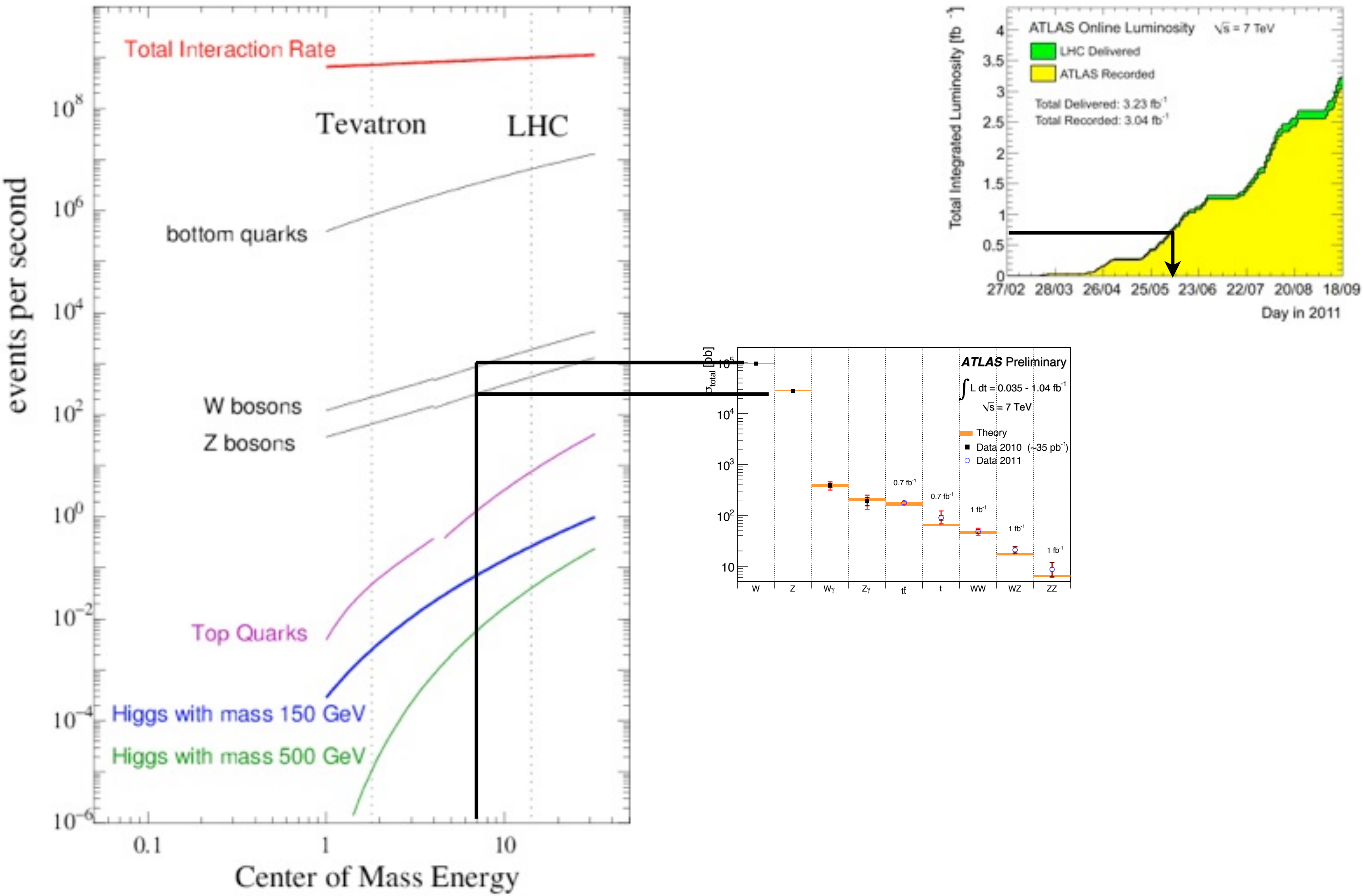
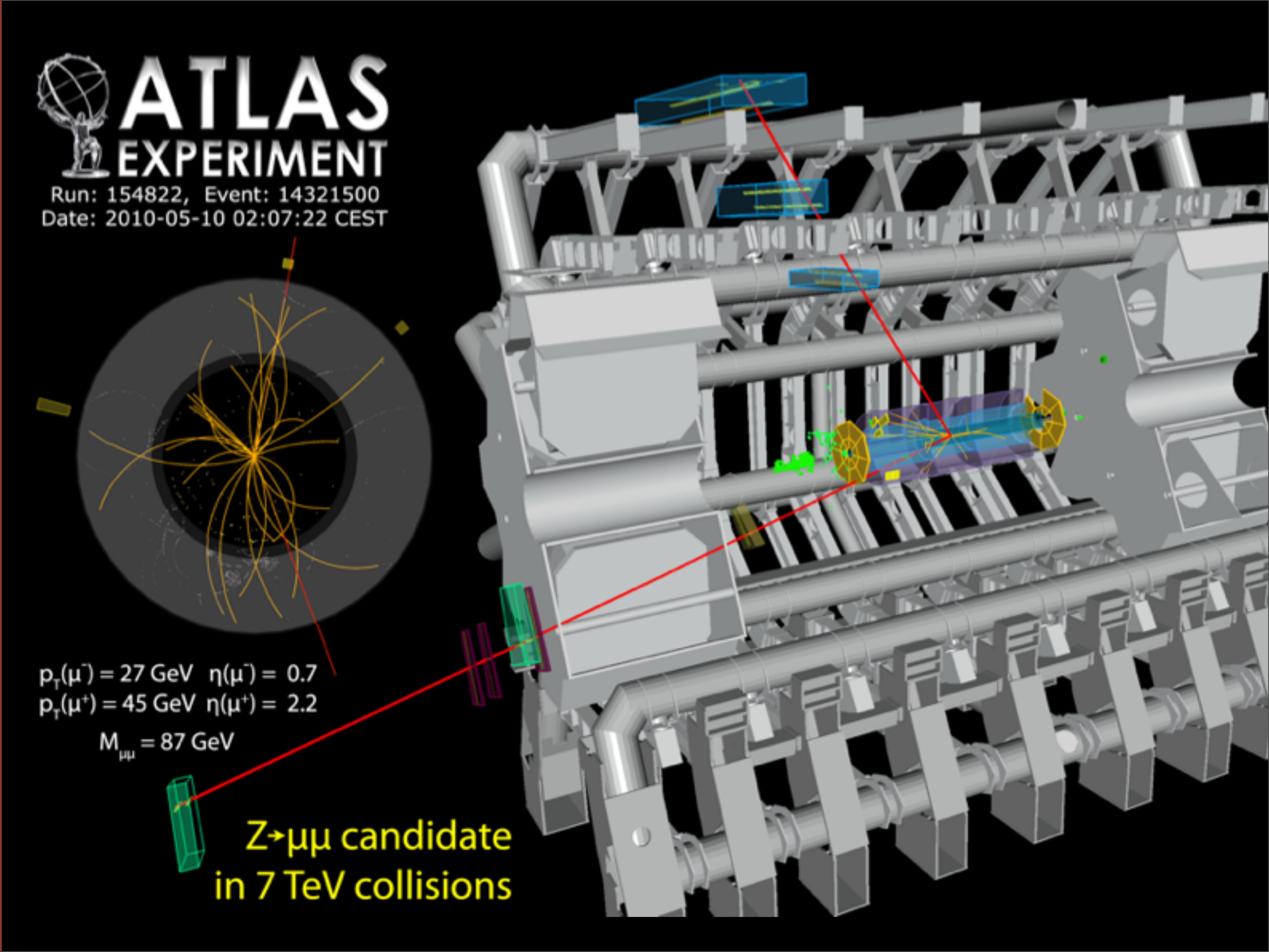
Run Number: 162620, Event Number: 16060241
Date: 2010-08-24 19:45:23 CEST

The steady march of progress




ATLAS EXPERIMENT
Run: 154822, Event: 14321500
Date: 2010-05-10 02:07:22 CEST
$p_T(\mu^-) = 27$ GeV $\eta(\mu^-) = 0.7$
$p_T(\mu^+) = 45$ GeV $\eta(\mu^+) = 2.2$
$M_{\mu\mu} = 87$ GeV
Z→μμ candidate
in 7 TeV collisions

The steady march of progress

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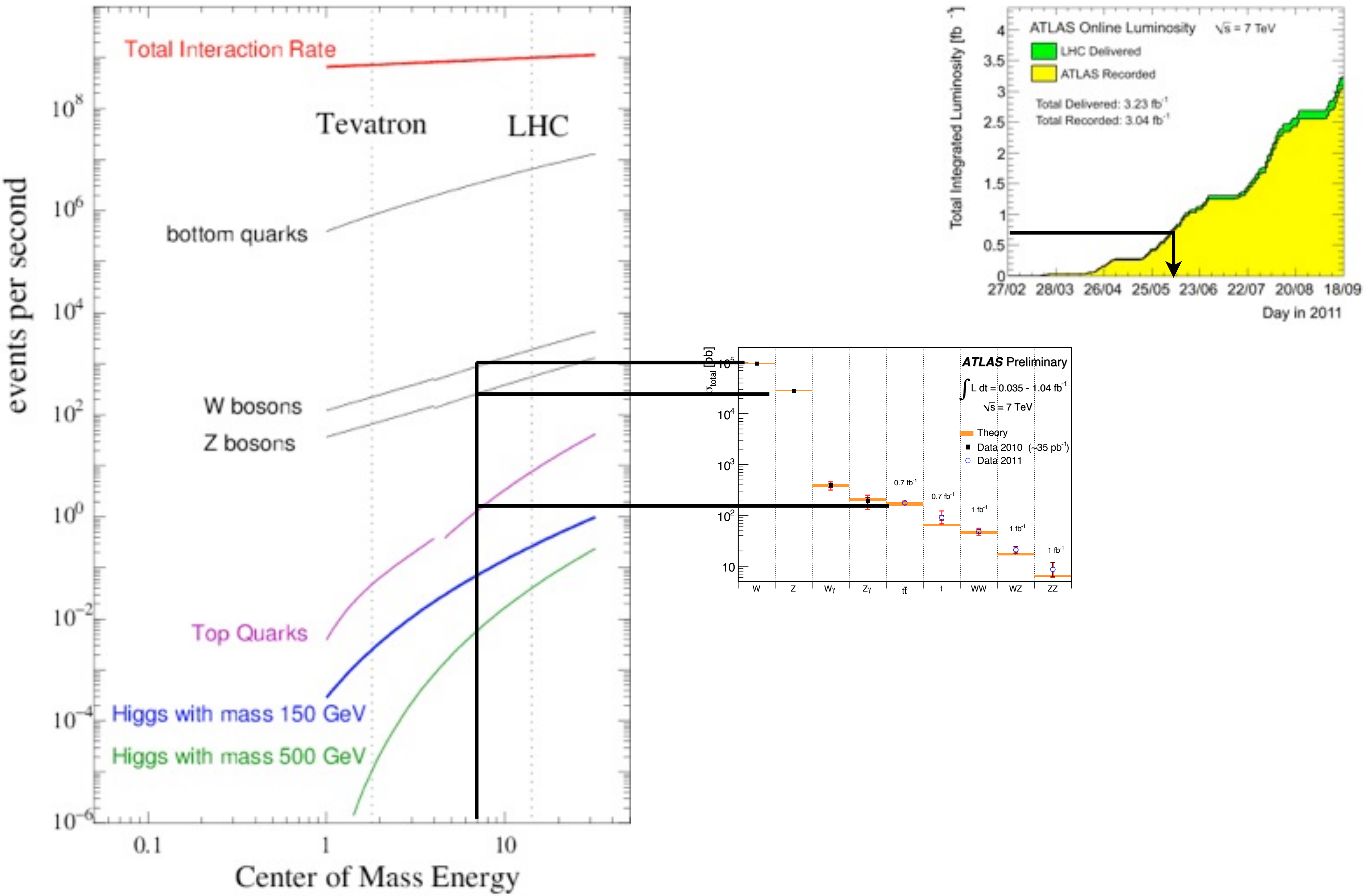
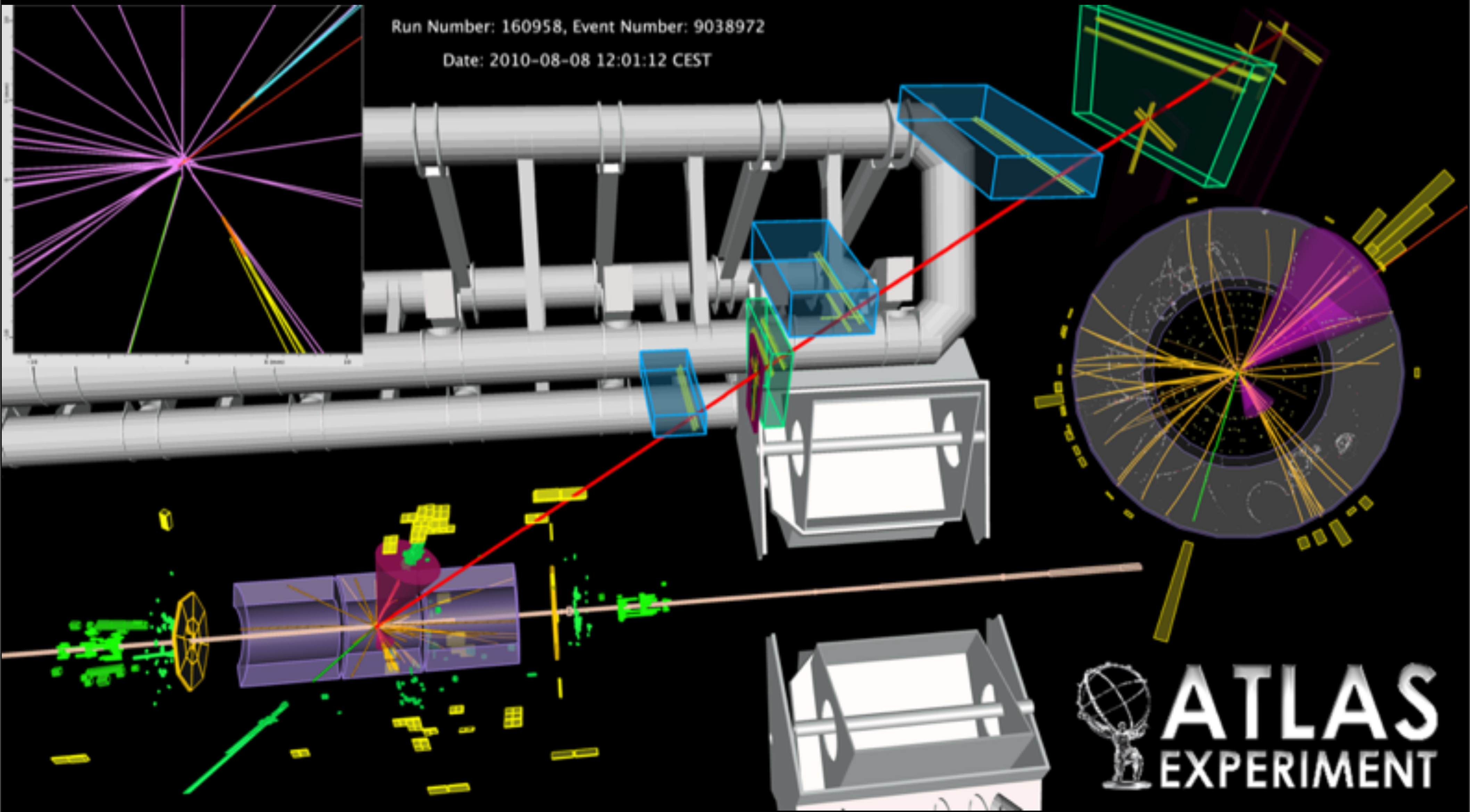


Top quark pair decaying to bb eμ $E_{T,miss}$



The steady march of progress

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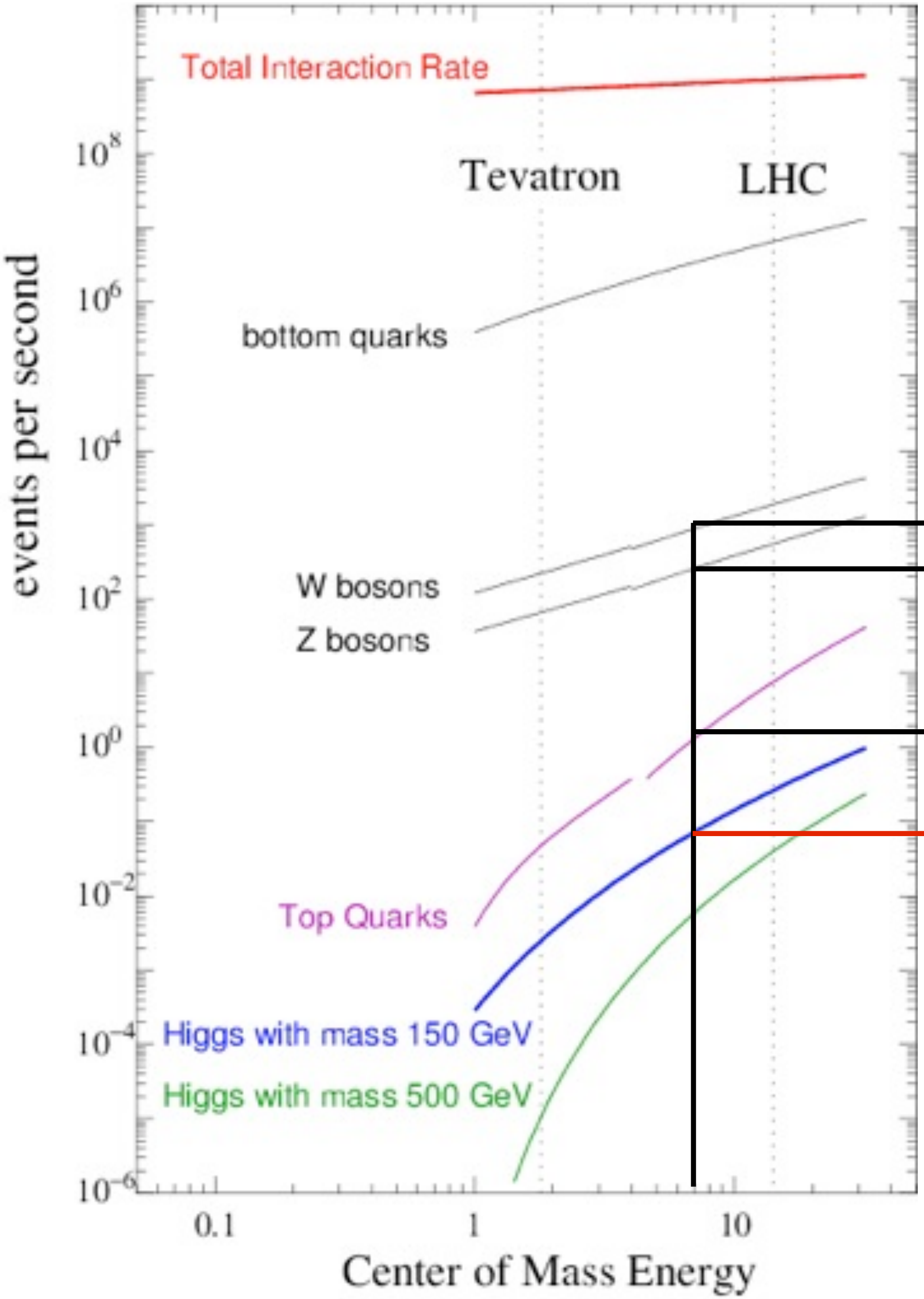
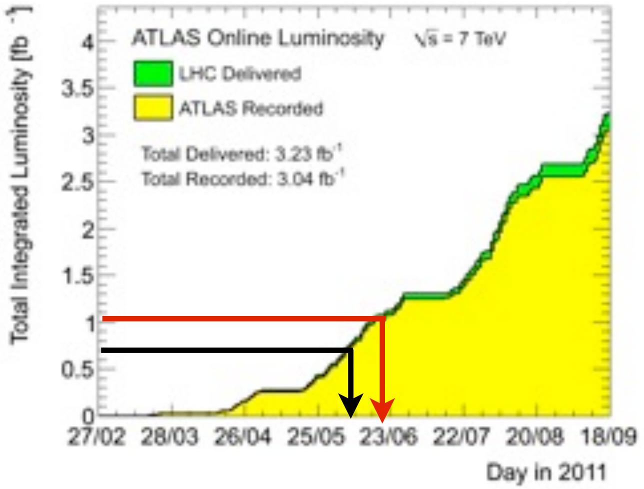
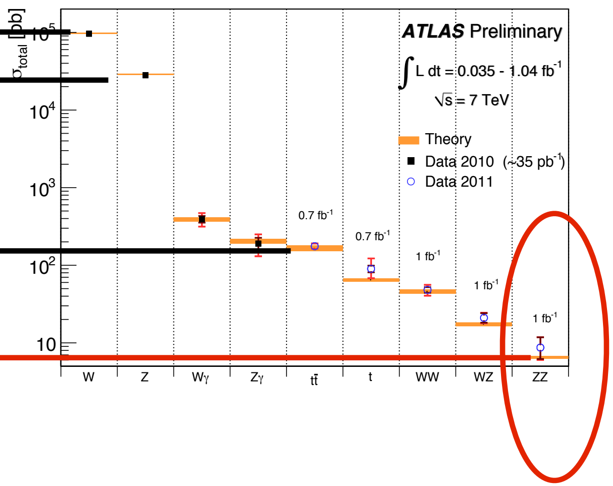
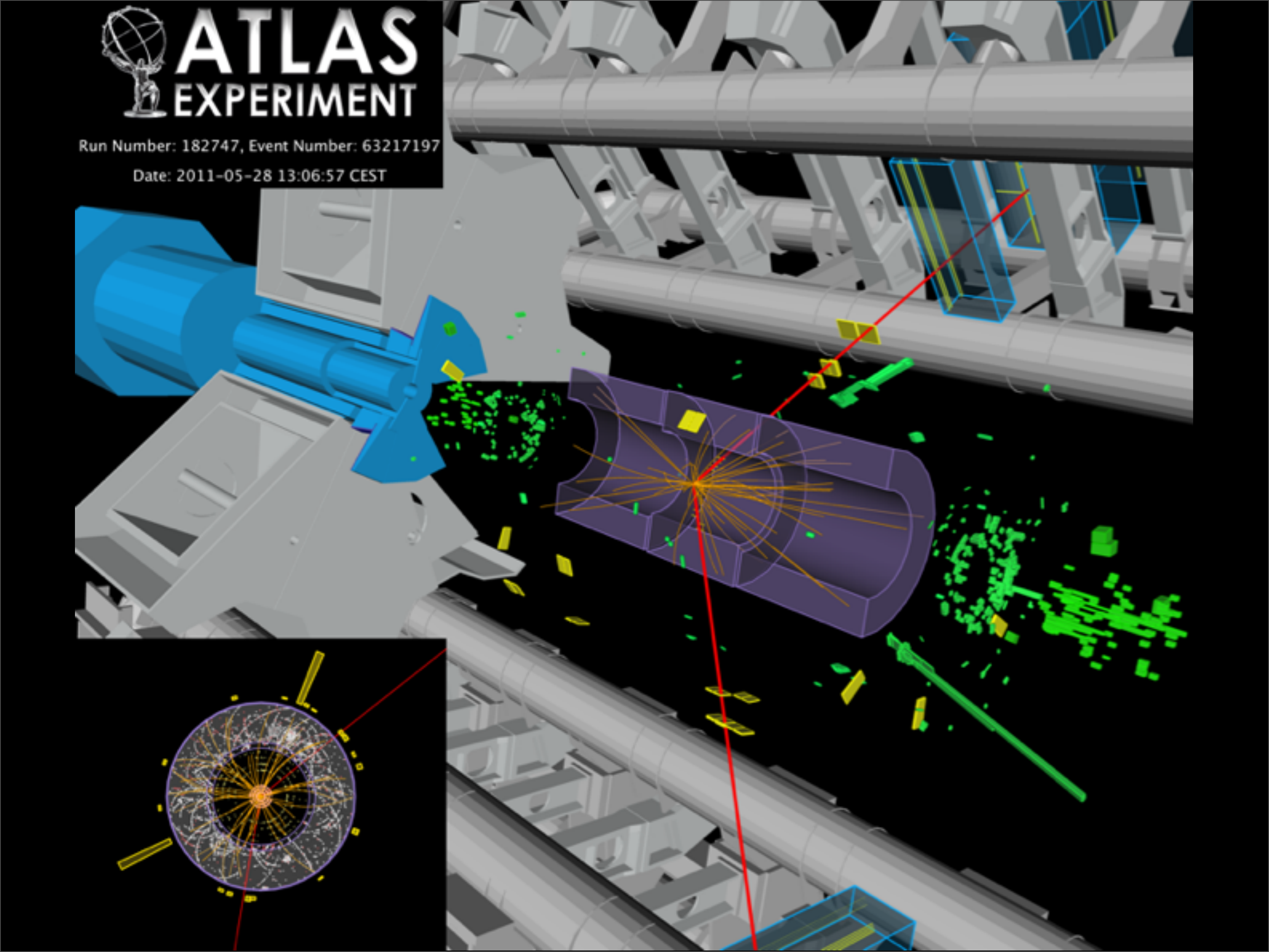
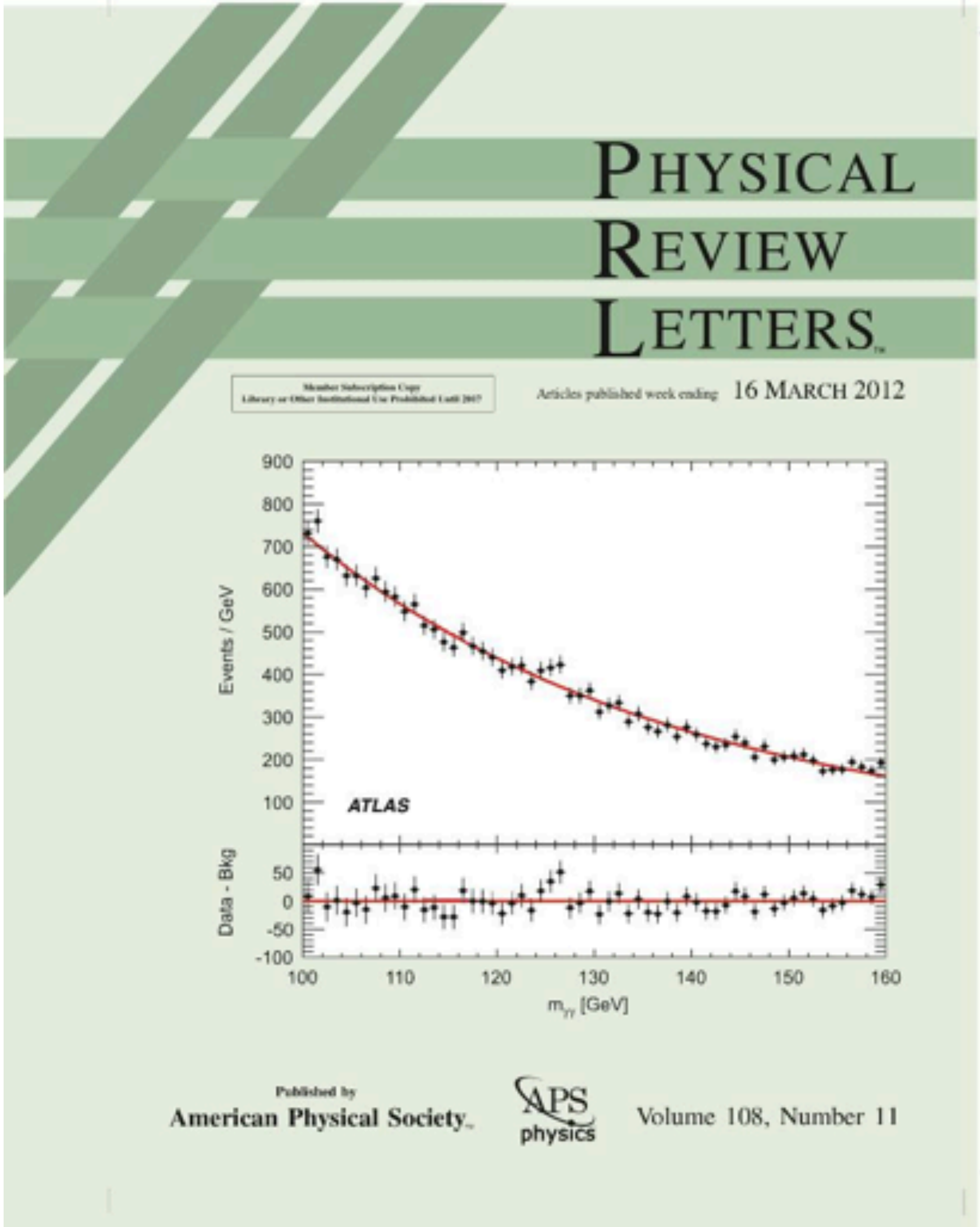


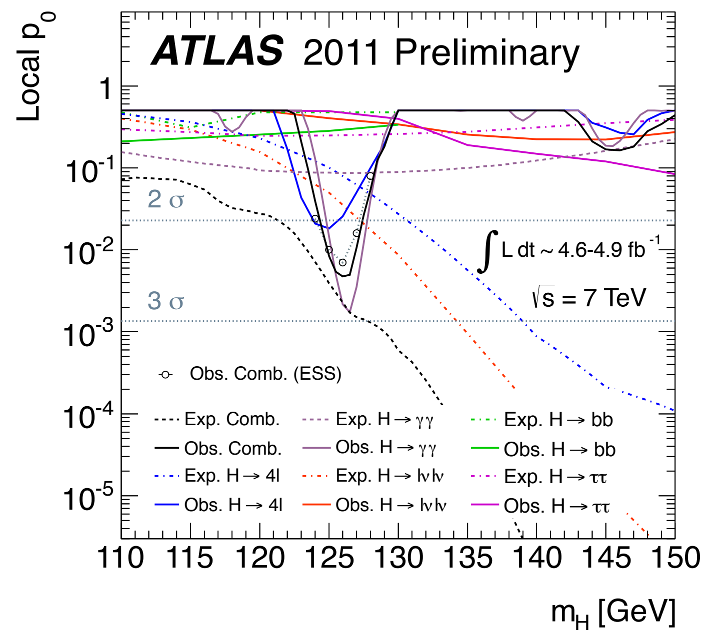


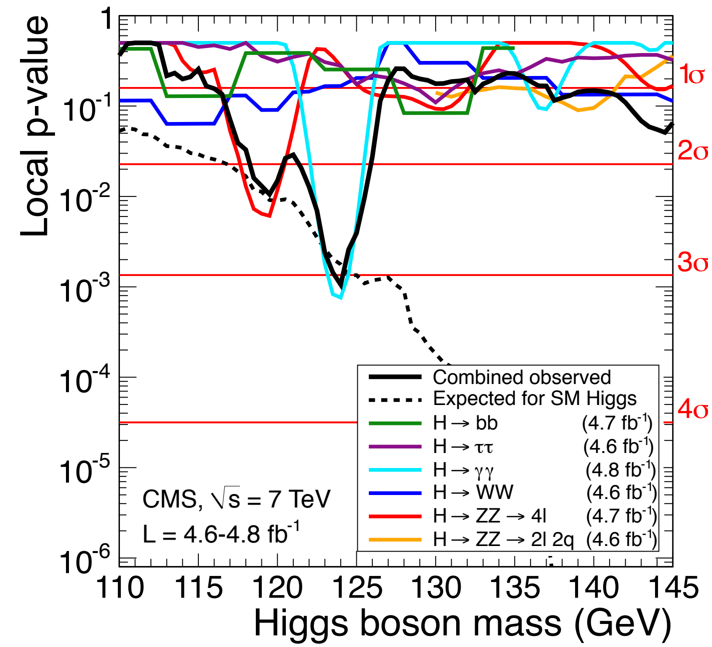



ATLAS
EXPERIMENT
Run Number: 182747, Event Number: 63217197
Date: 2011-05-28 13:06:57 CEST

News of the day





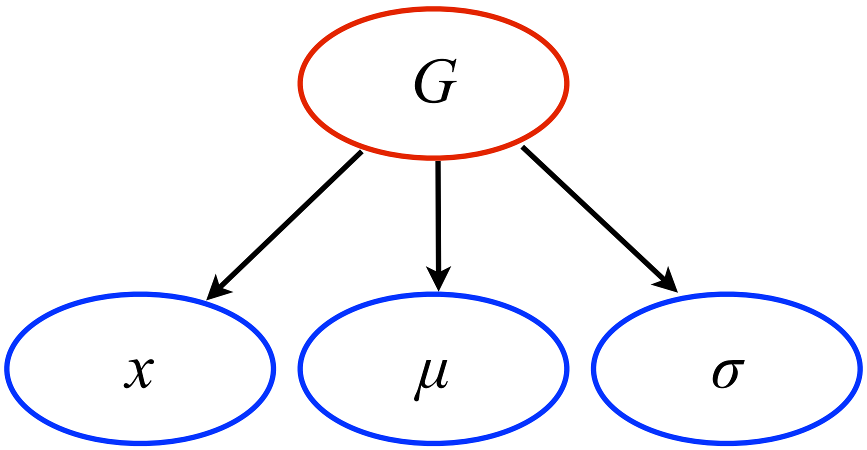


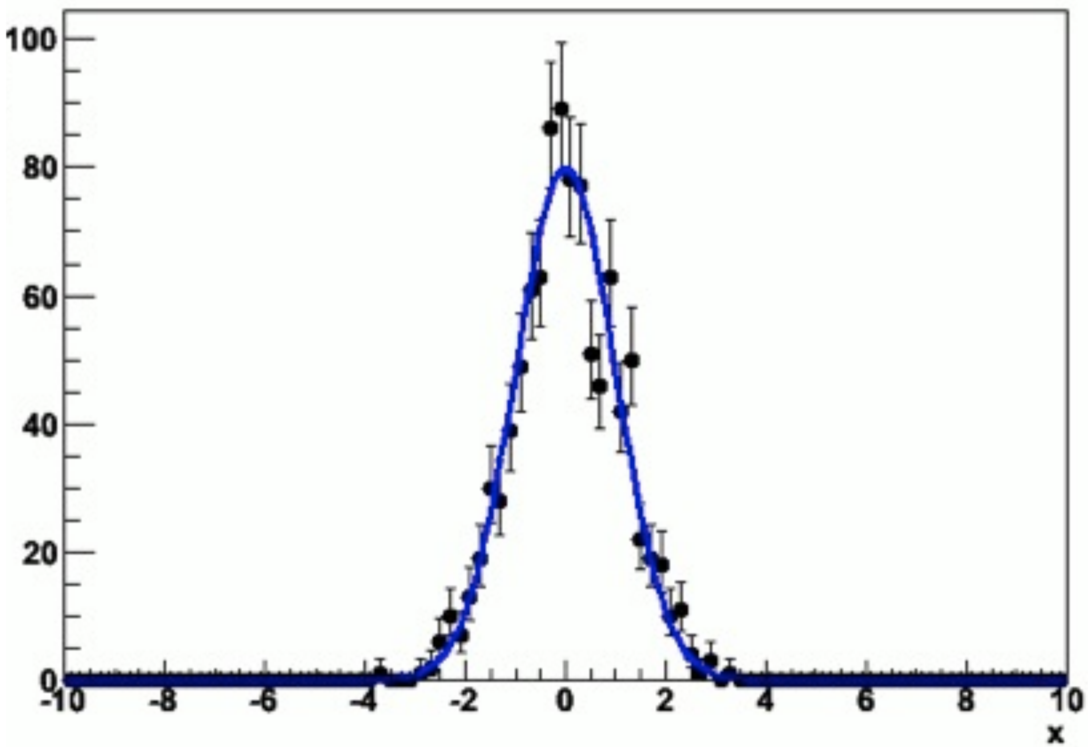
Modeling

Visualizing probability models

I will represent PDFs graphically as below (directed acyclic graph)

- eg. a Gaussian $G(x|\mu,\sigma)$ is parametrized by (μ,σ)
- every node is a real-valued function of the nodes below





Clearly related to Graphical Models, but not the focus here.

Marked Poisson Process

Channel: a subset of the data defined by some selection requirements.

- eg. all events with 4 electrons with energy > 10 GeV
- n: number of events observed in the channel
- ν: number of events expected in the channel

Discriminating variable: a property of those events that can be measured and which helps discriminate the signal from background

- eg. the invariant mass of two particles
- $f(x)$: the p.d.f. of the discriminating variable x

$$\mathcal{D} = \{x_1, \ldots, x_n\}$$

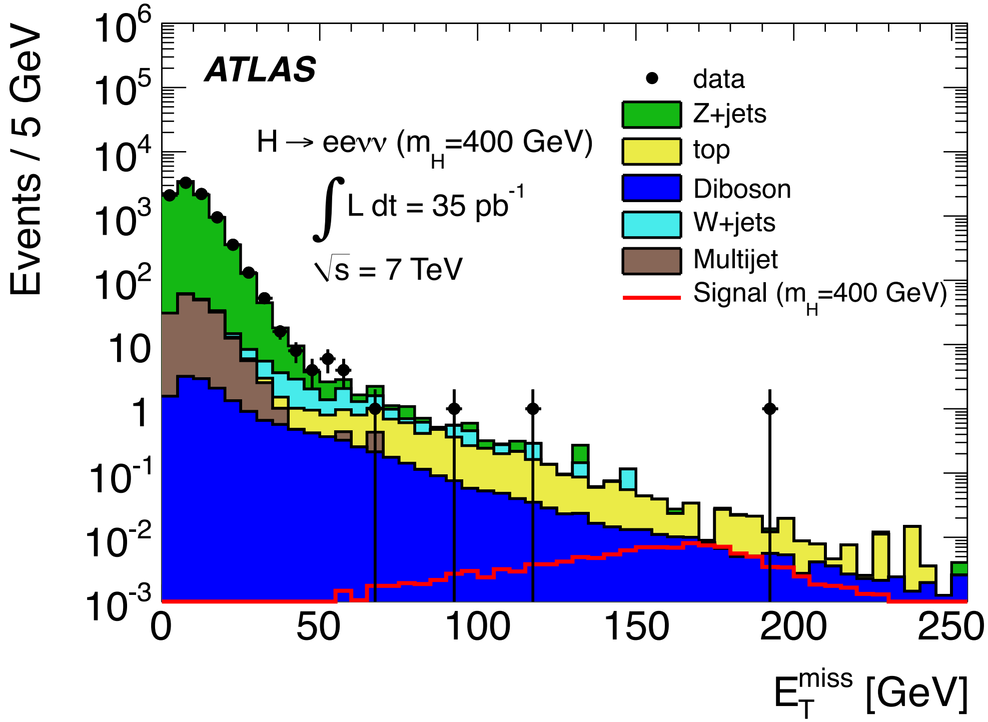
Marked Poisson Process:

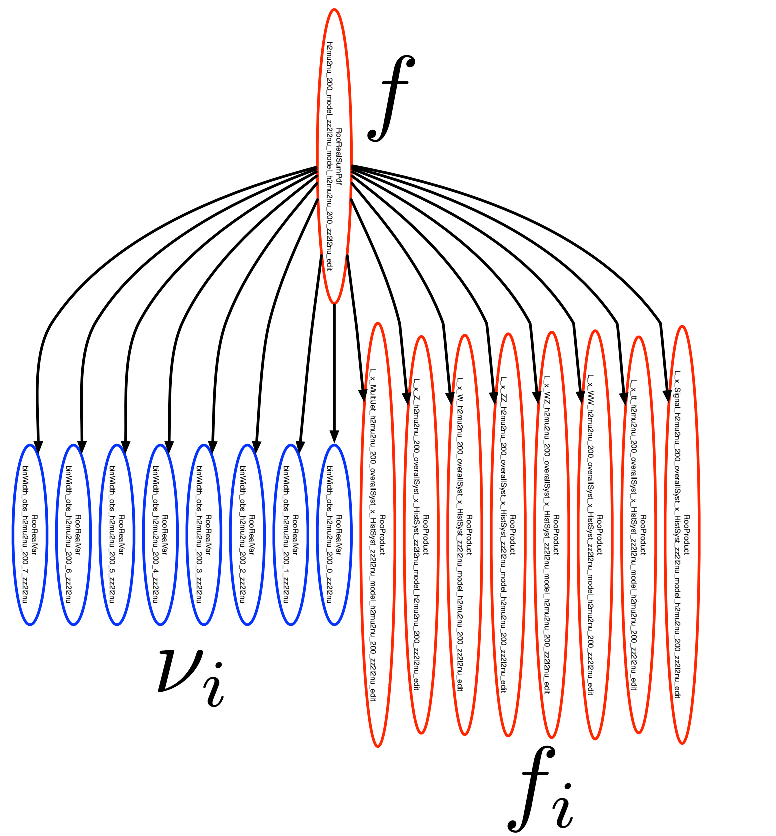
$$\mathbf{f}(\mathcal{D}|\nu) = \mathrm{Pois}(n|\nu) \prod_{e=1}^{n} f(x_e)$$

Mixture model

Total distribution is a mixture model with components corresponding to various signal and background interactions

$$f(x) = \frac{1}{\nu} \sum_{i \in \mathrm{interactions}} \nu_i \, f_i(x) \,, \quad \nu = \sum_{i \in \mathrm{interactions}} \nu_k$$





Parametrizing the model $\boldsymbol{\alpha} = (\mu, \boldsymbol{\theta})$

Parameters of interest (*μ*): parameters of the theory that modify the rates and shapes of the distributions, eg.

- the mass of a hypothesized particle
- the "signal strength" μ=0 no signal, μ=1 predicted signal rate

Nuisance parameters (*θ* or α_p**):** associated to uncertainty in:

- response of the detector (calibration)
- phenomenological model of interaction in non-perturbative regime

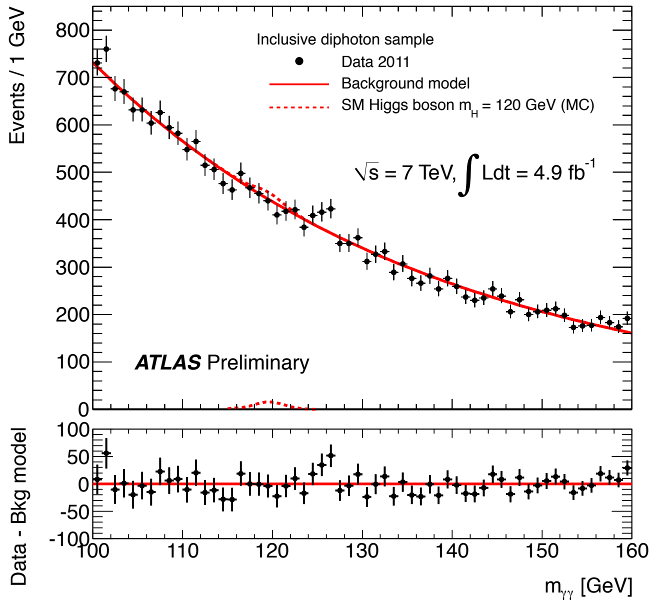
Lead to a parametrized model: $\nu \to \nu(\boldsymbol{\alpha}), f(x) \to f(x|\boldsymbol{\alpha})$

$$\mathbf{f}(\mathcal{D}|\boldsymbol{\alpha}) = \mathrm{Pois}(n|\boldsymbol{\alpha}) \prod_{e=1}^{n} f(x_e|\boldsymbol{\alpha})$$

Parametric vs. non-parametric

The densities $f_i(x|\boldsymbol{\alpha})$ for signal and background interactions are typically estimated either parametrically or non-parametrically

- in both cases simulated events are used for validation



Parametric modeling:

- simple exponential or polynomial
- parameters have no obvious interpretation

Non-Parametric modeling:

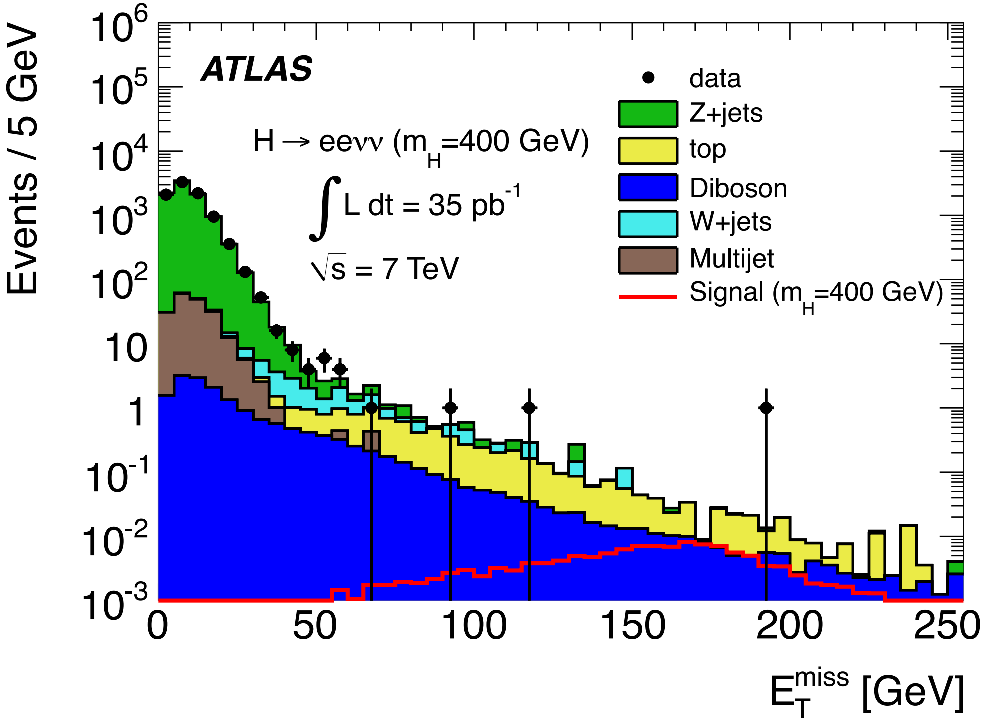
- densities estimated with histograms or kernel estimation based on simulated samples with specific settings $\{\{x_{ij}\}, \boldsymbol{\alpha}_j\}$
- parameters have clear physical interpretation
- difficulty is forming a parametrized model in many dimensions
- see contribution from Radford Neal, PhyStat 2007 [limited to likelihood]

Incorporating Systematic Effects

Center for Cosmology and Particle Physics

Tabulate effect of individual variations of sources of systematic uncertainty

- typically one at a time evaluated at nominal and "± 1 σ"
- use some form of interpolation to parametrize p^{th} variation in terms of **nuisance parameter** α_p



	Z+jets	top	Diboson	...
syst 1				
syst 2				
...				

$$\mathbf{f}(\mathcal{D}|\boldsymbol{\alpha}) = \mathrm{Pois}(n|\boldsymbol{\alpha}) \prod_{e=1}^{n} f(x_e|\boldsymbol{\alpha})$$

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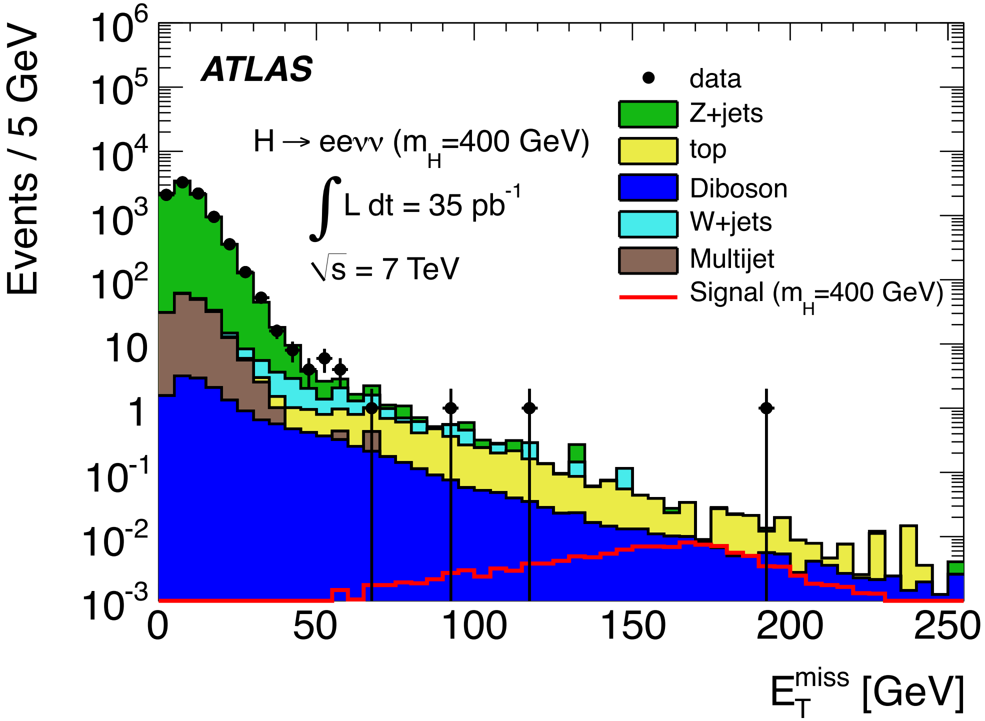

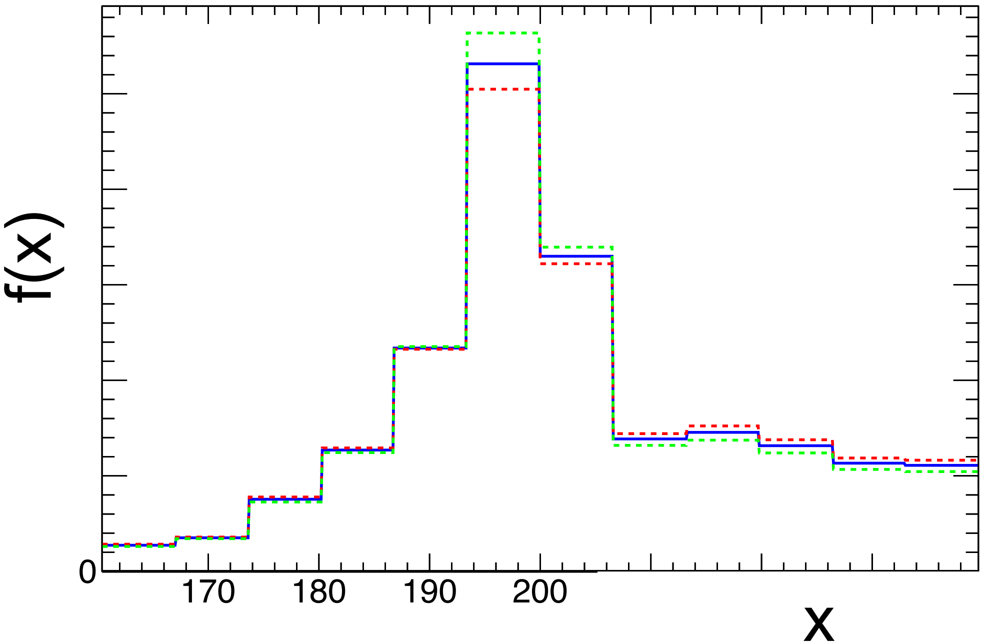

$$\mathbf{f}(\mathcal{D}|\boldsymbol{\alpha}) = \mathrm{Pois}(n|\boldsymbol{\alpha}) \prod_{e=1}^{n} f(x_e|\boldsymbol{\alpha})$$

Incorporating Systematic Effects

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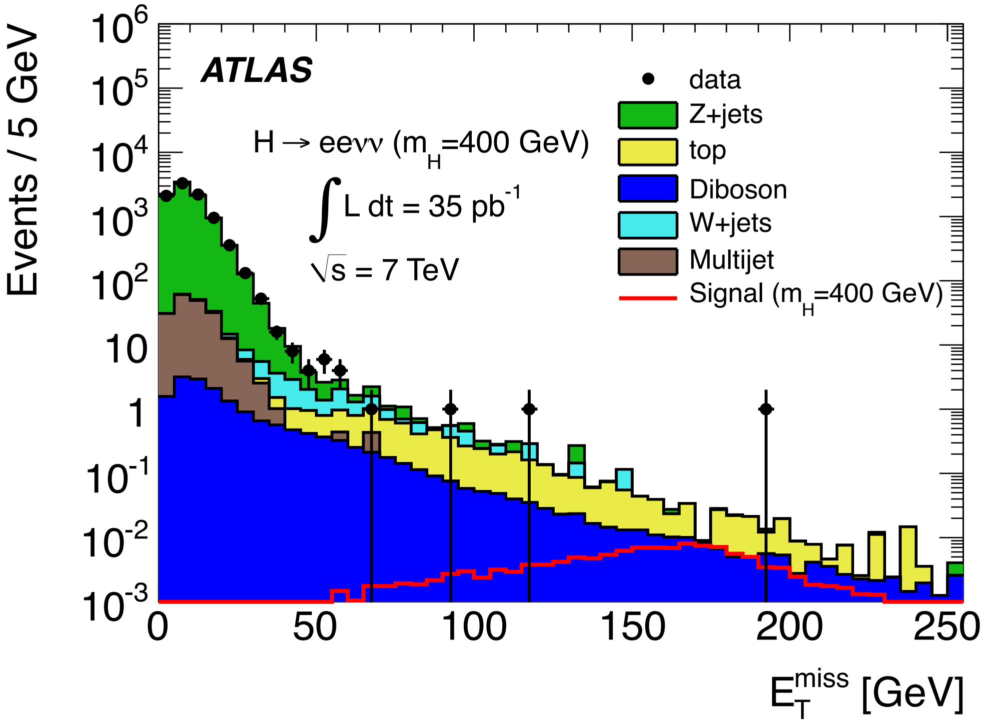
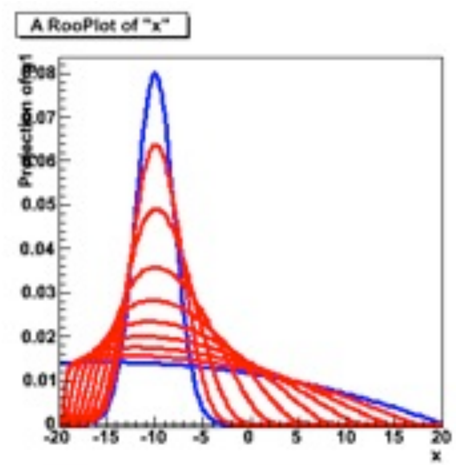
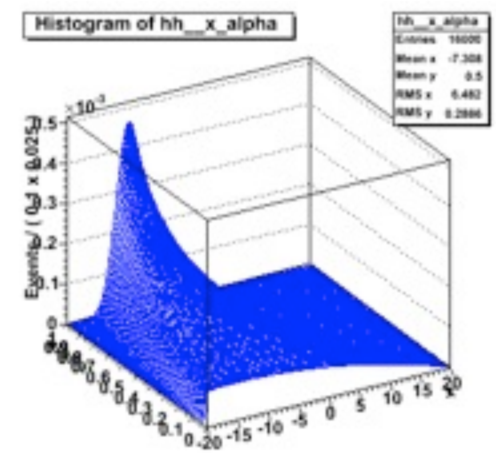
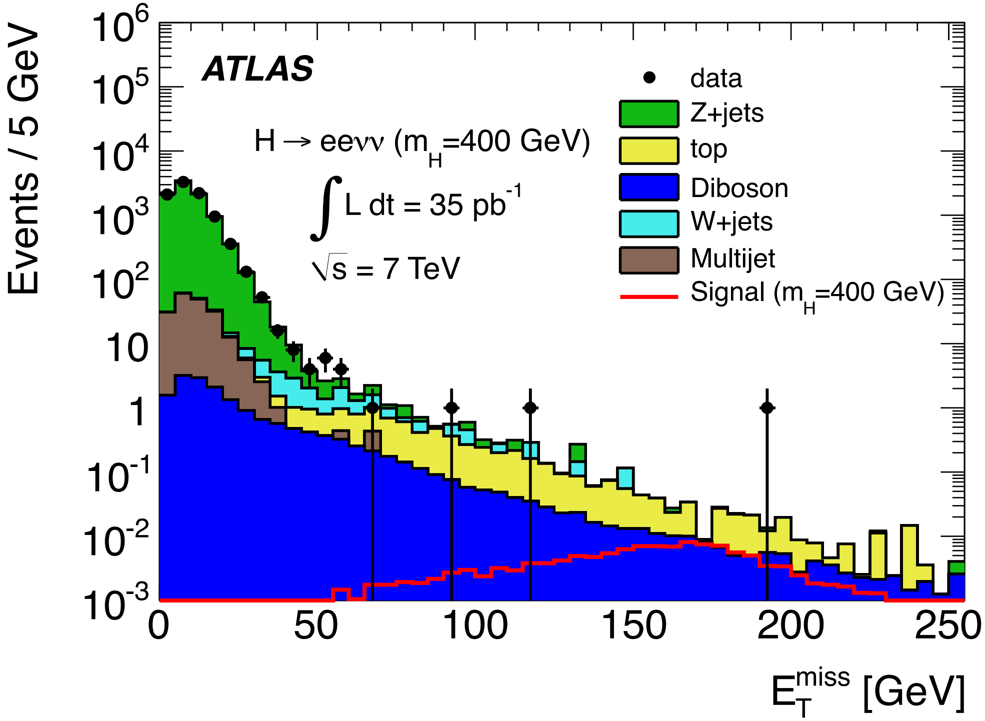


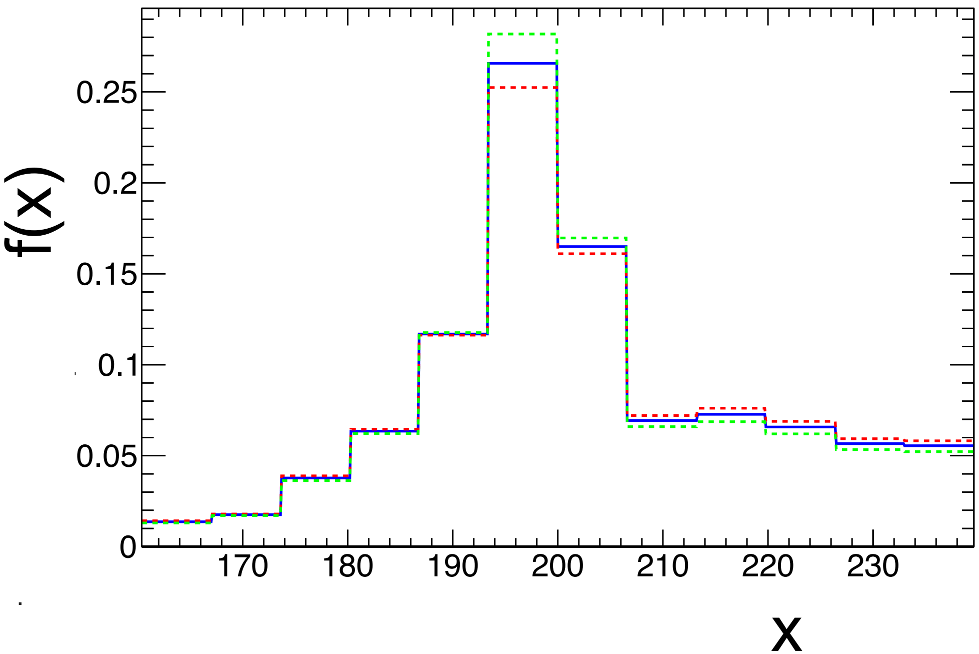


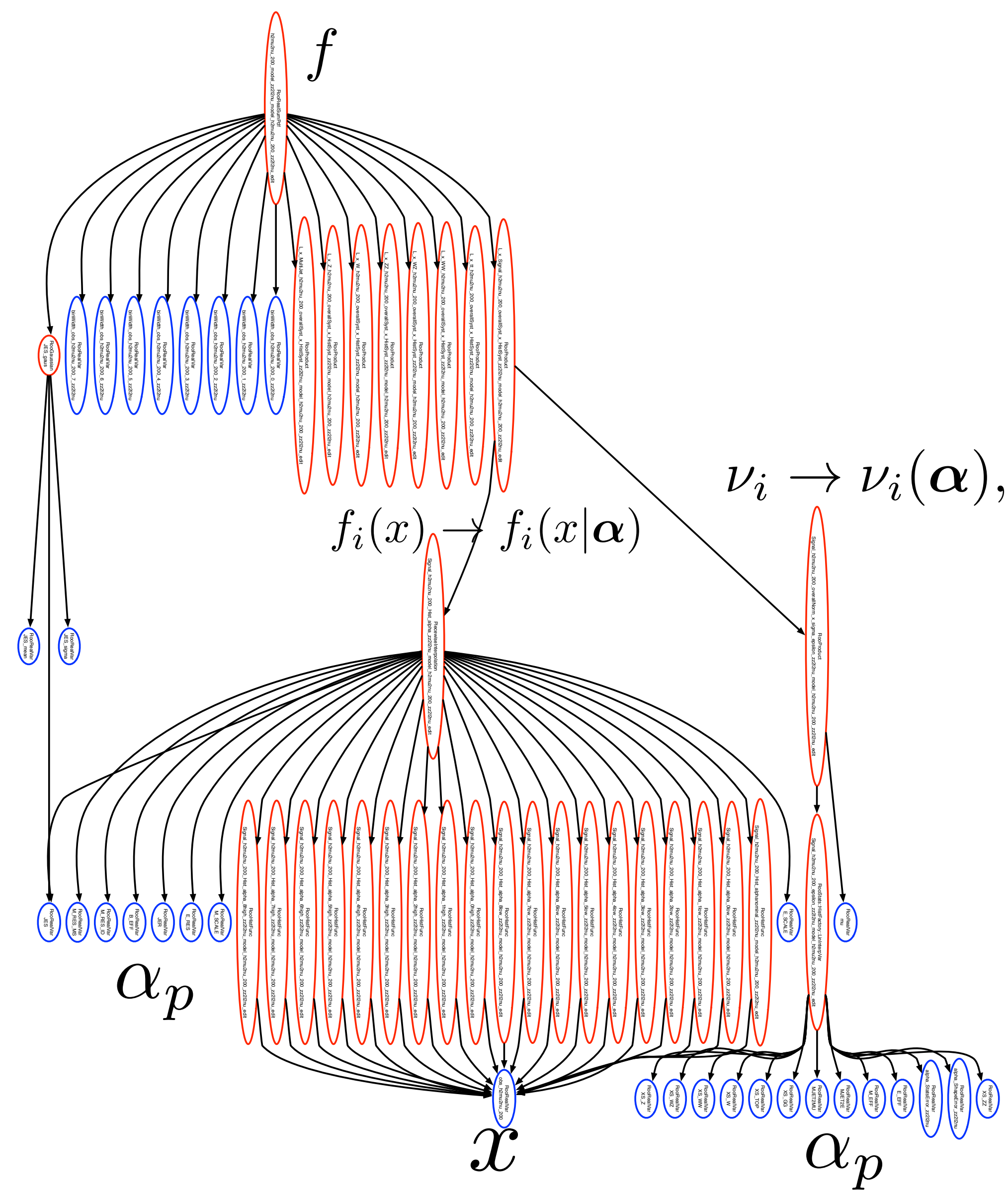


$$\mathbf{f}(\mathcal{D}|\boldsymbol{\alpha}) = \mathrm{Pois}(n|\boldsymbol{\alpha}) \prod_{e=1}^{n} f(x_e|\boldsymbol{\alpha})$$

Visualizing the model for one channel

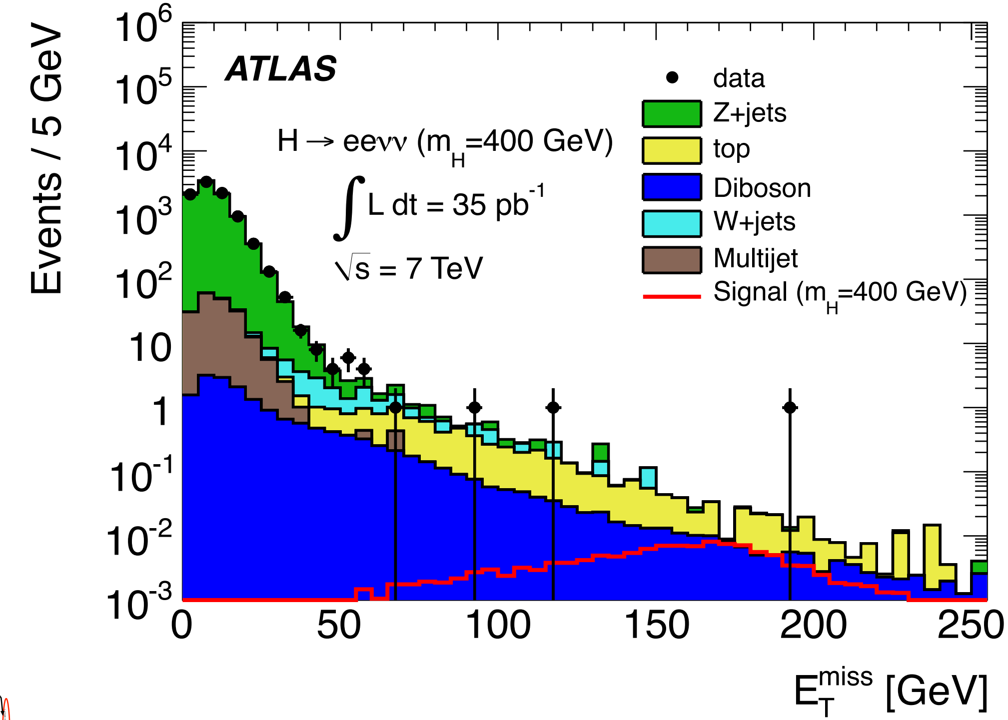


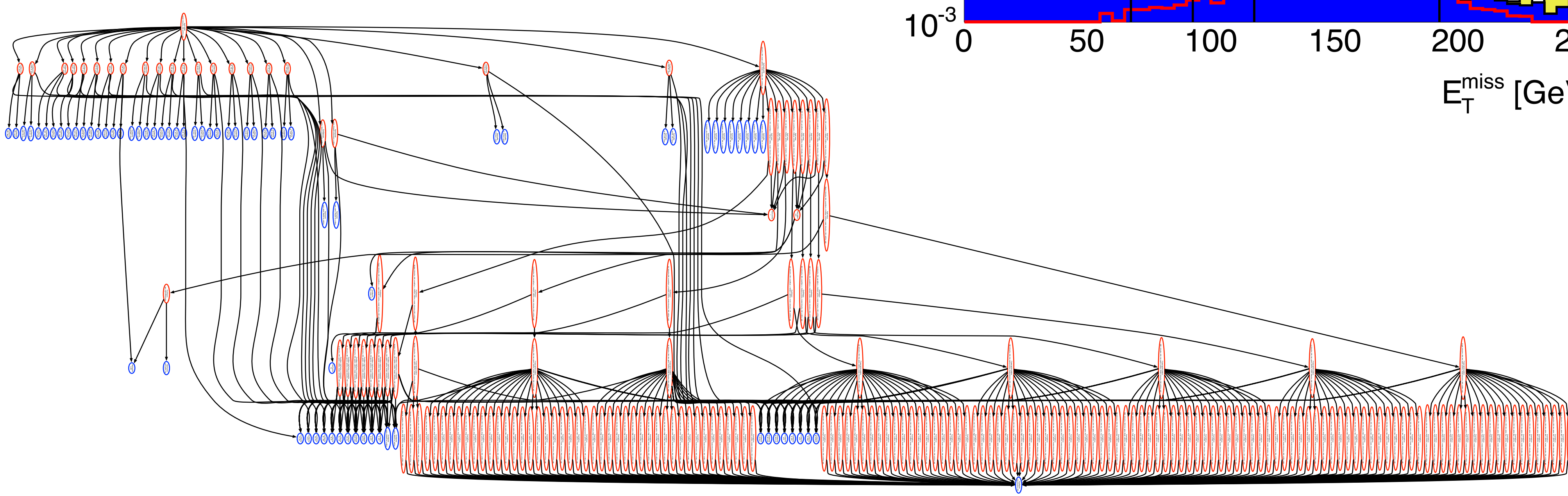




Visualizing the model for one channel

After parametrizing each component of the mixture model, the pdf for a single channel might look like this





Simultaneous multi-channel model

Simultaneous Multi-Channel Model: Several disjoint regions of the data are modeled simultaneously. Identification of common parameters across many channels requires coordination between groups such that meaning of the parameters are really the same.

$$\mathbf{f}_{\rm sim}(\mathcal{D}_{\rm sim}|\boldsymbol{\alpha}) = \prod_{c\in{\rm channels}} \left[{\rm Pois}(n_c|\nu_c(\boldsymbol{\alpha})) \prod_{e=1}^{n_c} f_c(x_{ce}|\boldsymbol{\alpha}) \right]$$

where $\mathcal{D}_{\rm sim} = \{\mathcal{D}_1, \ldots, \mathcal{D}_{c_{\rm max}}\}$

Control Regions: Some channels are not populated by signal processes, but are used to constrain the nuisance parameters

- convert ‘bad’ systematics into ‘good’ systematics
- Prototypical Example: “on/off” problem with unknown ν_b

$$\mathbf{f}(n,m|\mu,\nu_b) = \underbrace{{\rm Pois}(n|\mu+\nu_b)}_{\text{signal region}} \cdot \underbrace{{\rm Pois}(m|\tau\nu_b)}_{\text{control region}}$$

Constraint terms

Often detailed statistical model for auxiliary measurements that measure certain nuisance parameters are not available.

- one typically has MLE for α_p, denoted a_p and standard error

Constraint Terms: are idealized densities for the MLE.

$$f_p(a_p|\alpha_p) \quad \text{for} \quad p \in \mathbb{S}$$

- common choices are Gaussian, Poisson, and log-normal
- New: careful to write constraint term a frequentist way.
- Previously: $\pi(\alpha_p|a_p) = f_p(a_p|\alpha_p)\eta(\alpha_p)$ with uniform η

Simultaneous Multi-Channel Model with constraints:

$$\mathbf{f}_{\rm tot}(\mathcal{D}_{\rm sim},\mathcal{G}|\boldsymbol{\alpha}) = \prod_{c\in\text{channels}}\left[\text{Pois}(n_c|\nu_c(\boldsymbol{\alpha}))\prod_{e=1}^{n_c} f_c(x_{ce}|\boldsymbol{\alpha})\right]\cdot\prod_{p\in\mathbb{S}} f_p(a_p|\alpha_p)$$

where $\mathcal{D}_{\rm sim} = \{\mathcal{D}_1,\ldots,\mathcal{D}_{c_{\max}}\}$, $\mathcal{G} = \{a_p\}$ for $p \in \mathbb{S}$

Constraint terms

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Often detailed statistical model for auxiliary measurements that measure certain nuisance parameters are not available.

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Simultaneous Multi-Channel Model with constraints:

$$\mathbf{f}_{\text{tot}}(\mathcal{D}_{\text{sim}}, \mathcal{G}|\boldsymbol{\alpha}) = \prod_{c \in \text{channels}} \left[\text{Pois}(n_c|\nu_c(\boldsymbol{\alpha})) \prod_{e=1}^{n_c} f_c(x_{ce}|\boldsymbol{\alpha}) \right] \cdot \prod_{p \in \mathbb{S}} f_p(a_p|\alpha_p)$$

where $\mathcal{D}_{\text{sim}} = \{\mathcal{D}_1, \ldots, \mathcal{D}_{c_{\max}}\}$, $\boxed{\mathcal{G} = \{a_p\}}$ for $p \in \mathbb{S}$

Combined ATLAS Higgs Search for 2011

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State of the art: Our most recent combined Higgs search includes 70 disjoint channels and >100 nuisance parameters

- Models for individual channels come from about 11 sub-groups performing dedicated searches for specific Higgs decay modes
- In addition low-level performance groups provide tools for evaluating systematic effects and corresponding constraint terms

Higgs Decay	Subsequent Decay	Additional Sub-Channels	m_H Range	L [fb^{-1}]
$H \to \gamma\gamma$	–	9 sub-channels ($p_{T_t} \otimes \eta_\gamma \otimes$ conversion)	110-150	4.9
$H \to ZZ$	$\ell\ell\ell'\ell'$	$\{4e, 2e2\mu, 2\mu 2e, 4\mu\}$	110-600	4.8
	$\ell\ell\nu\nu$	$\{ee, \mu\mu\} \otimes$ {low pile-up, high pile-up}	200-280-600	4.7
	$\ell\ell qq$	{b-tagged, untagged}	200-300-600	4.7
$H \to WW$	$\ell\nu\ell\nu$	$\{ee, e\mu, \mu\mu\} \otimes$ {0-jet, 1-jet, VBF}	110-300-600	4.7
	$\ell\nu q\overline{q'}$	$\{e, \mu\} \otimes$ {0-jet, 1-jet}	300-600	4.7
$H \to \tau^+\tau^-$	$\ell\ell 4\nu$	$\{e\mu\} \otimes$ {0-jet} $\oplus$ {1-jet, VBF, VH}	110-150	4.7
	$\ell\tau_{\text{had}}3\nu$	$\{e, \mu\} \otimes$ {0-jet} $\otimes$ $\{E_T^{\text{miss}} \gtrsim 20$ GeV$\}$ $\oplus$ $\{e, \mu\} \otimes$ {1-jet, VBF}	110-150	4.7
	$\tau_{\text{had}}\tau_{\text{had}}2\nu$	{1-jet}	110-150	4.7
$VH \to b\overline{b}$	$Z \to \nu\overline{\nu}$	$E_T^{\text{miss}} \in \{120-160, 160-200, \geq 200 \text{ GeV}\}$	110-130	4.6
	$W \to \ell\nu$	$p_T^W \in \{< 50, 50-100, 100-200, \geq 200 \text{ GeV}\}$	110-130	4.7
	$Z \to \ell\ell$	$p_T^Z \in \{< 50, 50-100, 100-200, \geq 200 \text{ GeV}\}$	110-130	4.7

Visualizing the combined model

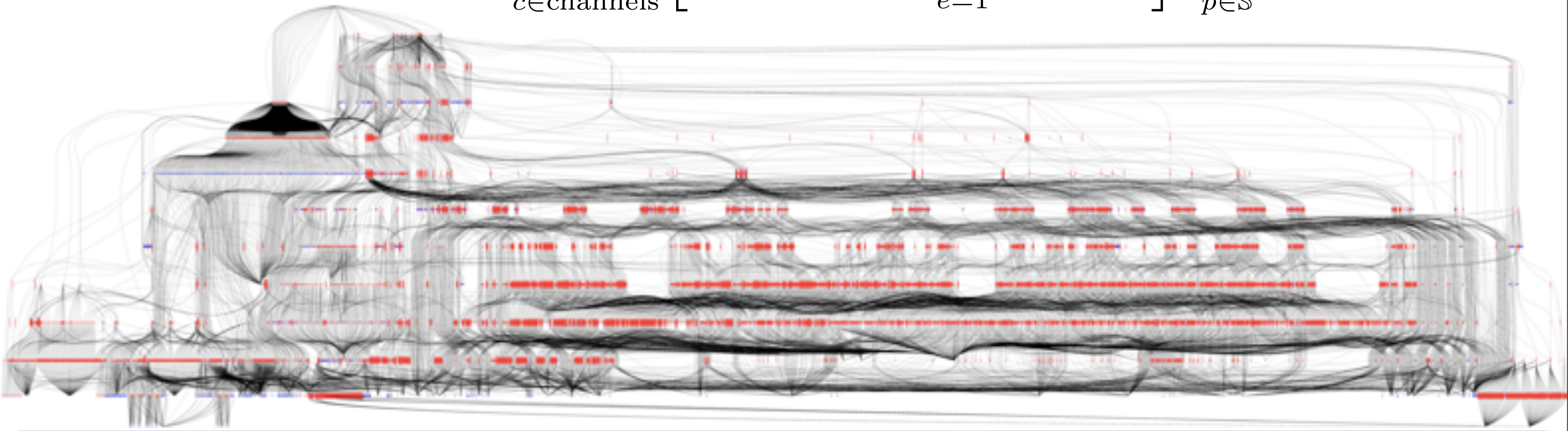
Center for
Cosmology and
Particle Physics

State of the art: Our most recent combined Higgs search includes 70 disjoint channels and >100 nuisance parameters

RooFit / RooStats: is the modeling language (C++) which provides technologies for collaborative modeling

- provides technology to publish likelihood functions digitally
- and more, it's the full model so we can also generate pseudo-data

$$\mathbf{f}_{\rm tot}(\mathcal{D}_{\rm sim}, \mathcal{G}|\boldsymbol{\alpha}) = \prod_{c\in\rm channels}\left[\mathrm{Pois}(n_c|\nu_c(\boldsymbol{\alpha}))\prod_{e=1}^{n_c} f_c(x_{ce}|\boldsymbol{\alpha})\right]\cdot\prod_{p\in\mathbb{S}} f_p(a_p|\alpha_p)$$



Statistical tests

Types of statistical tests

Remember, we haven't discovered a new particle since 1995 and we have been searching for the Higgs boson for many decades

- particle physics takes a very conservative approach towards discovery and exclusion

Particle physicists are also 'frequentist dinosaurs' in that we worry a lot about coverage of our confidence intervals etc.

- We also use Bayesian techniques, but there is a general dislike for assigning prior probabilities to theoretical parameters like the mass of the Higgs boson or priors on models (ie. prior probability that the Higgs boson exists at all)

With the full model $\mathbf{f}_{\rm tot}(\mathcal{D}_{\rm sim}, \mathcal{G} | \mu, \boldsymbol{\theta})$ we can easily make likelihood function and Bayesia posterior, converse is not true.

Timeline of the test statistic

In 1990's the LEP experiments had small systematic uncertainties, so the search for the Higgs was essentially a simple hypothesis test

- use of likelihood ratio following from Neyman-Pearson lemma

$$\frac{L(\mu = 1)}{L(\mu = 0)}$$

In 2000's the Tevatron experiments moved towards

- profiling nuisance parameters, but testing null against specific alternative

$$\frac{L(\mu = 1, \hat{\boldsymbol{\theta}}(\mu = 1))}{L(\mu = 0, \hat{\boldsymbol{\theta}}(\mu = 0))}$$

Recently, the LHC experiments moved towards profile likelihood ratio

- Wilks's theorem: asymptotic distribution is independent of $\boldsymbol{\theta}$

$$\lambda(\mu) = \frac{L(\mu, \hat{\hat{\boldsymbol{\theta}}}(\mu))}{L(\hat{\mu}, \hat{\boldsymbol{\theta}})}$$

Hybrid resampling / profile construction

Until recently, practice was to compute p-values with a Hybrid Bayesian-Frequentist procedure in which the nuisance parameters were marginalized with respect to some prior $\pi(\boldsymbol{\theta})$

$$p = \int_{t_{\rm obs}}^{\infty} dt \left[\int d\boldsymbol{\theta} f(t|\mu, \boldsymbol{\theta}) \pi(\boldsymbol{\theta}) \right]$$

Frequentist p-values should be calculated for a specific point in $\mu,\boldsymbol{\theta}$

Confidence interval for parameter of interest, consists of μ such that $P(\lambda(\mu) < c_\alpha | \mu, \boldsymbol{\theta}) < \alpha$ for all $\boldsymbol{\theta}$

- checking condition for all $\boldsymbol{\theta}$ is computationally challenging!
- approximate by considering only $\hat{\theta}(\mu)$

C. Chuang and T. L. Lai. Hybrid resampling methods for confidence intervals. *Statist. Sinica*, 10:1–50, 2000. http://www3.stat.sinica.edu.tw/statistica/oldpdf/A10n11.pdf.

Matthew Walker Bodhisattva Sen and Michael Woodroofe. On the unified method with nuisance parameters. *Statist. Sinica*, 19:301–314., 2009. http://www3.stat.sinica.edu.tw/statistica/oldpdf/A19n116.pdf.

Choice of test statistic

An important property of the profile likelihood ratio test statistic is that under certain conditions its asymptotic distribution is known

- Wilks's theorem [Ann. Math. Statist. 9 (1938)]:

$$f(-2\ln\lambda(\mu)|\mu,\boldsymbol{\theta})$$ chi-square, independent of θ

- Wald's theorem [Trans. Amer. Math. Soc. V54, No. 3 (1943)]

$$f(-2\ln\lambda(\mu)|\mu',\boldsymbol{\theta})$$ non-central chi-square

- with non-centrality parameter given by the Fisher information matrix

$$V_{ij}^{-1} = -E\left[\frac{\partial^2 \ln L}{\partial\theta_i\partial\theta_j}\right]$$

Our parameter of interest μ bounded physically to be non-negative

- fairly easy to incorporate the effect of boundary

Perhaps a clever trick for calculating the expected Fisher information matrix

- "Asimov data" is an artificial dataset with special properties
- observed Fisher information on Asimov = expected Fisher information

The Asimov dataset

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The expected Fisher information is not trivial to compute for an arbitrary model

$$\left(\mathcal{I}\left(\theta\right)\right)_{i,j} = -\operatorname{E}\left[\left.\frac{\partial^2}{\partial\theta_i\,\partial\theta_j}\ln f(X;\theta)\right|\theta\right].$$

In the case of a binned distribution, the expected Fisher information is identical to the observed Fisher information calculated with an artificial dataset, referred to as the Asimov data

- follows from fact that $\partial^2[\ln \mathrm{Pois}(n|\theta)]/\partial\theta^2$ is linear in *n*
- The bin contents for the Asimov data are exactly the expectation of the model (non-integer in general)

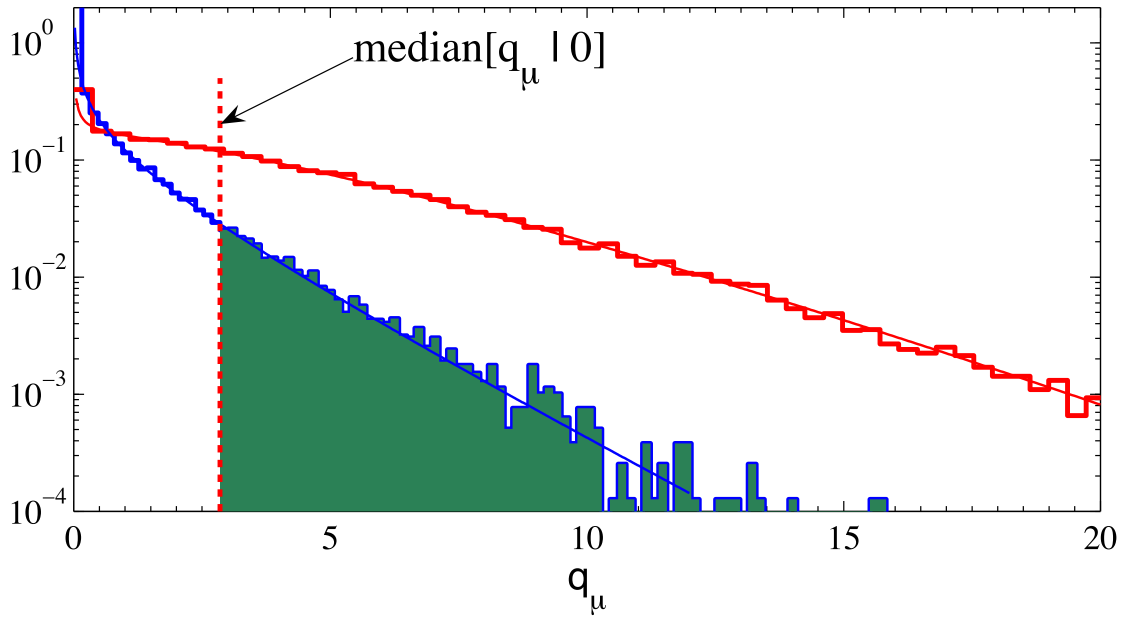
This is a very convenient numerical trick to calculate:

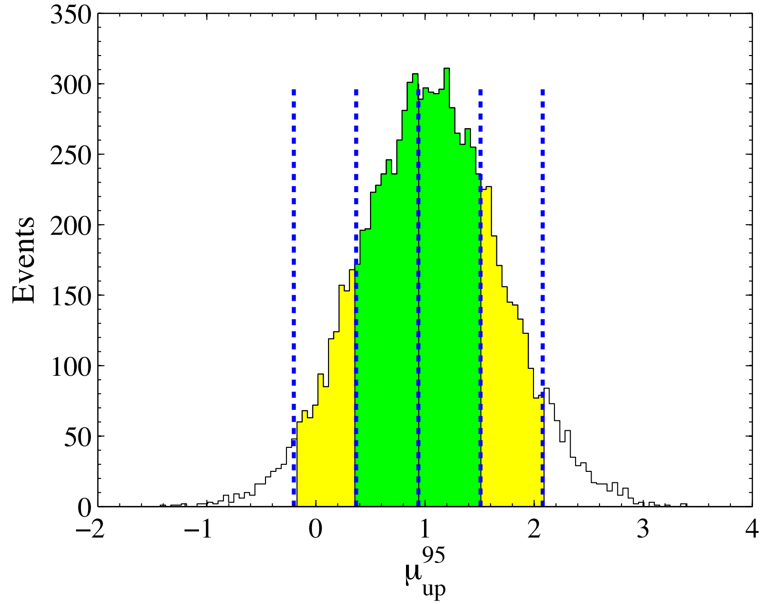
- expected Fisher information
- non-centrality parameter (Wald's theorem)
- Jeffreys's Prior (also helps with reference prior)
  - Recent progress in Reference analysis in particle physics:

L. Demortier, S. Jain, H. Prosper Phys.Rev. D82 (2010) 034002

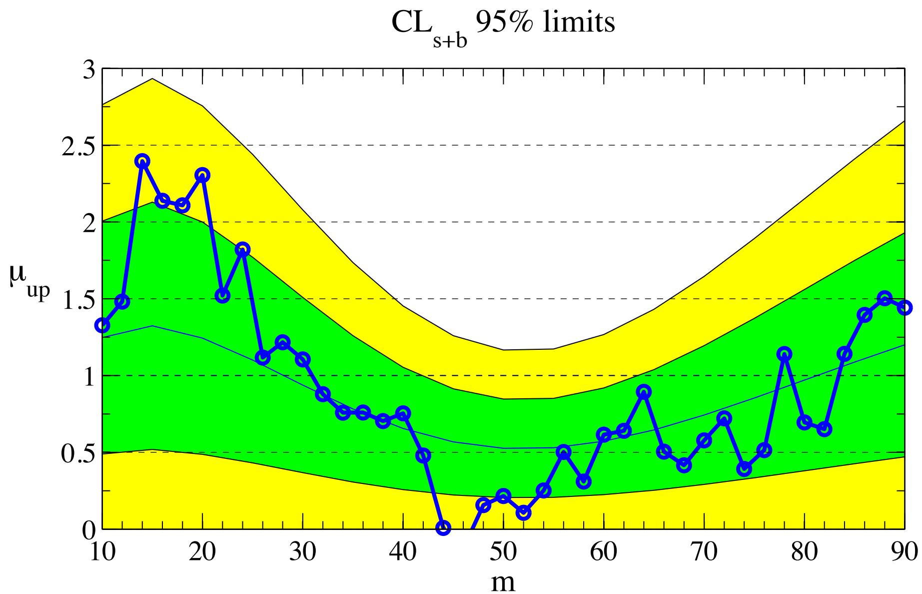
Comparison of asymptotic and ensembles

Compare asymptotic distributions to distributions obtained with large ensembles of pseudo-experiments generated with Monte Carlo techniques





This is a significant development as building this distribution from Monte Carlo approaches can take 100,000 CPU hours for Higgs search!

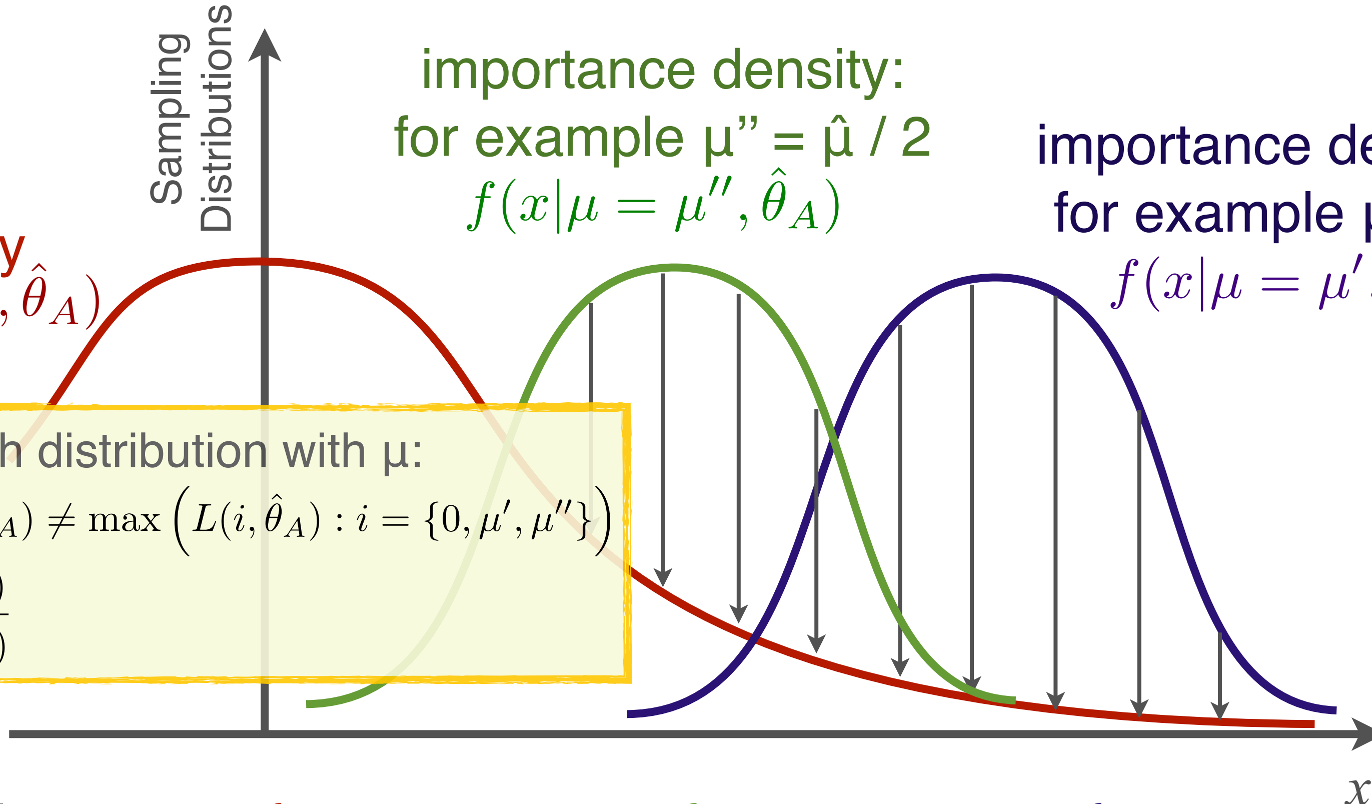


G. Cowan, KC, E. Gross, O. Vitells
Eur.Phys.J. C71 (2011) 1554
[arXiv:1007.1727]

Importance sampling

At Banff, M. Woodroofe presented an approach to importance sampling

- The importance density was based on a model averaged over some parameters, which is technically difficult for complex models
- since then, we



For toys in each distribution with μ:

$$\text{veto: if } L(\mu, \hat{\theta}_A) \neq \max\left(L(i, \hat{\theta}_A) : i = \{0, \mu', \mu''\}\right)$$

$$\text{weight} = \frac{L(0, \hat{\theta}_A)}{L(\mu, \hat{\theta}_A)}$$

merged distribution = toys from b-only + toys from imp dens 1 + toys from imp dens 2

Importance sampling

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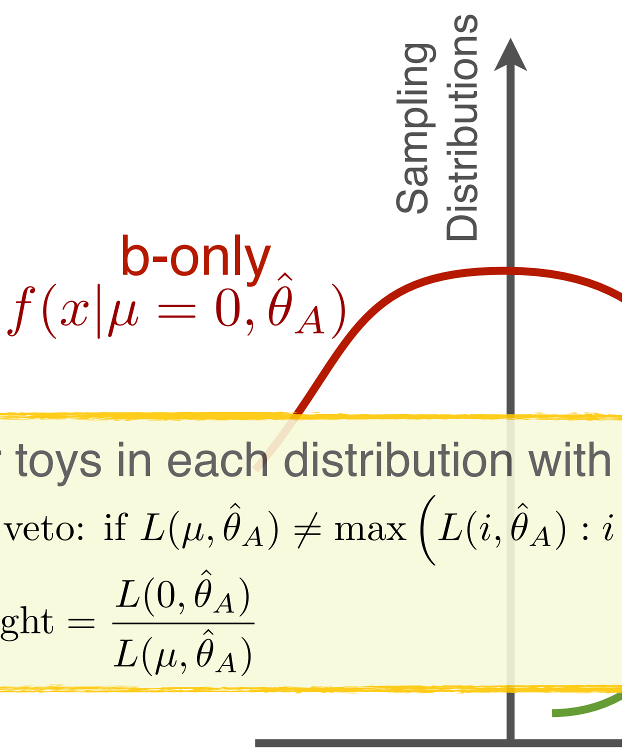


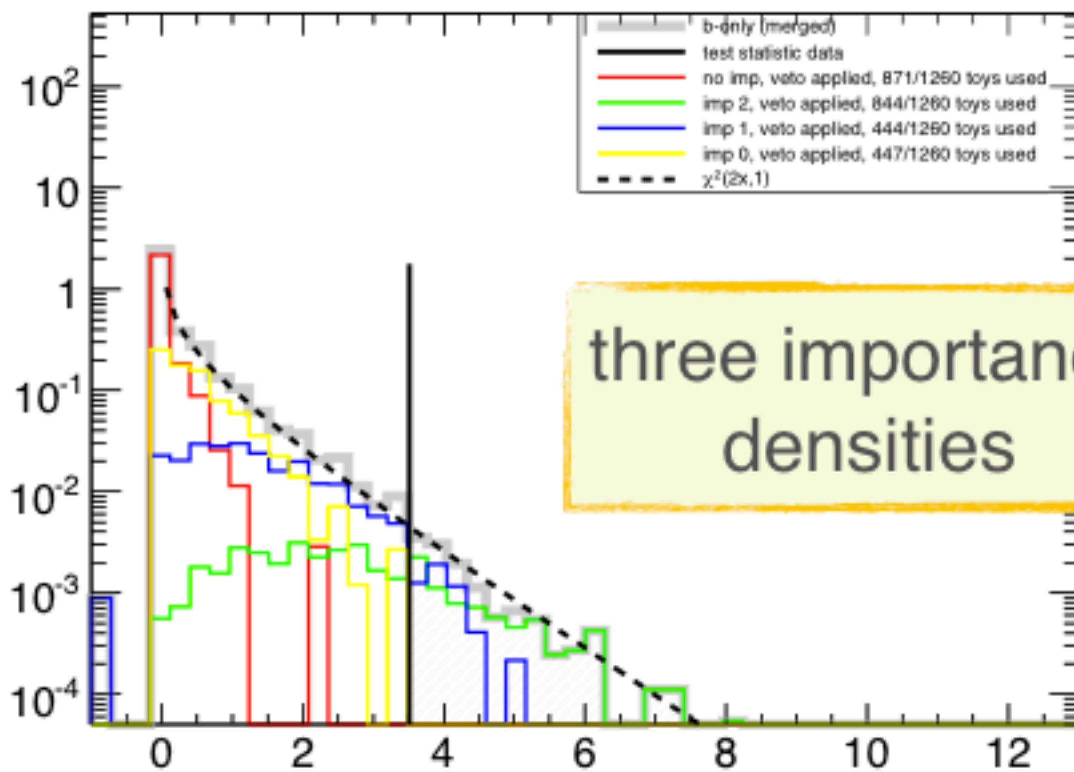
For toys in each distribution with

$$\text{veto: if } L(\mu,\hat{\theta}_A) \neq \max\Big(L(i,\hat{\theta}_A) : i$$

$$\text{weight} = \frac{L(0,\hat{\theta}_A)}{L(\mu,\hat{\theta}_A)}$$

merged distribution = toys from b-only

importance density:



three importance densities

Just to mention

Presence of physical boundary μ≥0 leads to complications in 1-sided upper-limits

- re-opened discussion on the "CLs method" and pushed it forward to a discussion on negatively biased relevant subsets, conditional coverage, and statistical evidence proposed by Allan Birnbaum.

R. D. Cousins. Negatively Biased Relevant Subsets Induced by the Most-Powerful One-Sided Upper Confidence Limits for a Bounded Physical Parameter. *ArXiv e-prints*, September 2011.

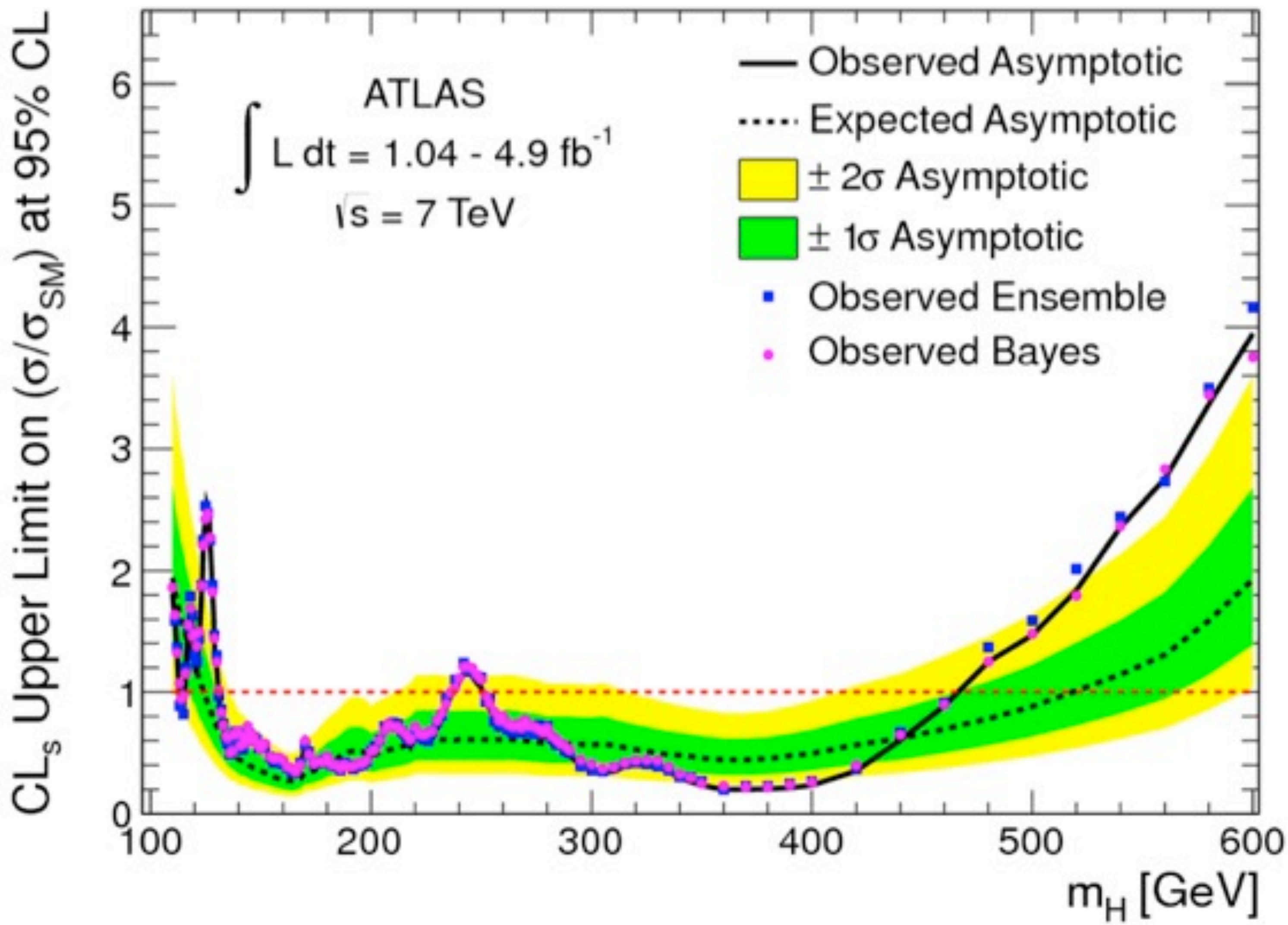
CLs upper limits. `http://en.wikipedia.org/wiki/CLs_upper_limits`.

Mark Mandelkern. Setting confidence intervals for bounded parameters. *Statistical Science*, 17(2):pp. 149–159, 2002.

G. Cowan, K. Cranmer, E. Gross, and O. Vitells. Power-Constrained Limits. *ArXiv e-prints*, May 2011.

Monte Carlo, asymptotics, vs. Bayesian

Here we see comparisons of explicit ensembles generated with Monte Carlo techniques, the asymptotic results, and Bayesian results using MCMC and nested sampling with a uniform prior on μ



Jeffreys's Prior

Jeffreys's Prior and Reference prior require expected Fisher information

$$\pi(\vec{\theta}) \propto \sqrt{\det \mathcal{I}\left(\vec{\theta}\right)}. \qquad\qquad (\mathcal{I}(\theta))_{i,j} = -\operatorname{E}\left[\frac{\partial^2}{\partial\theta_i\,\partial\theta_j}\ln f(X;\theta)\right|\theta\left.\vphantom{\frac{\partial^2}{\partial\theta_i}}\right].$$

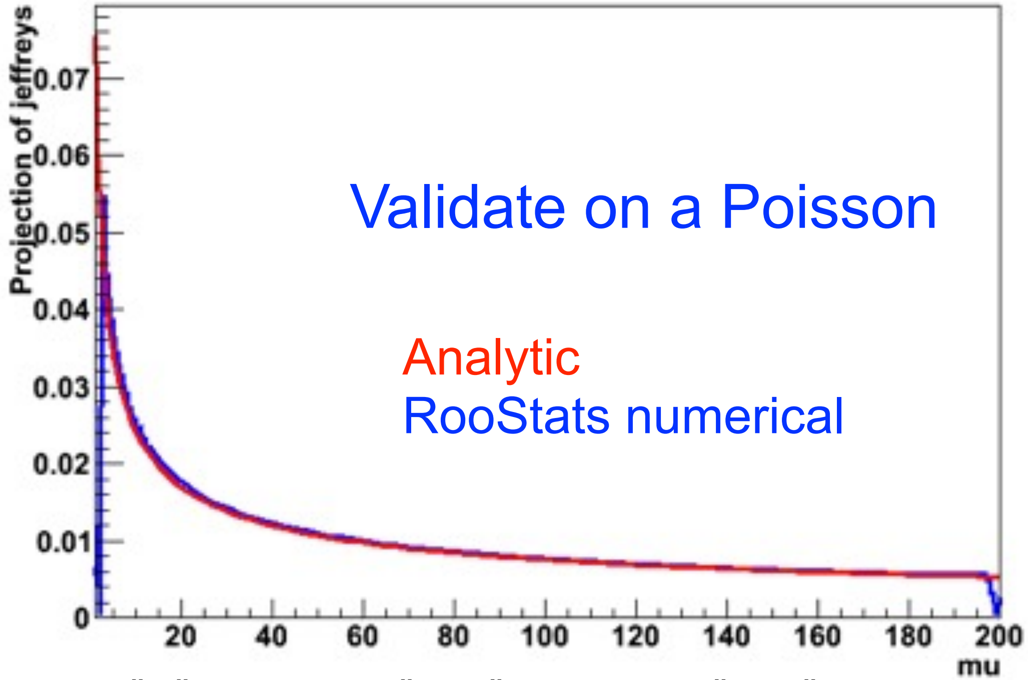
The Asimov data provides a fast, convenient way to calculate the Fisher Information

$$V_{jk}^{-1} = -E\left[\frac{\partial^2 \ln L}{\partial\theta_j \partial\theta_k}\right] = -\frac{\partial^2 \ln L_{\mathrm{A}}}{\partial\theta_j \partial\theta_k} = \sum_{i=1}^{N} \frac{\partial\nu_i}{\partial\theta_j}\frac{\partial\nu_i}{\partial\theta_k}\frac{1}{\nu_i} + \sum_{i=1}^{M} \frac{\partial u_i}{\partial\theta_j}\frac{\partial u_i}{\partial\theta_k}\frac{1}{u_i}$$

Use this as basis to calculate Jeffreys's prior for an arbitrary PDF!

```
RooWorkspace w("w");
w.factory("Uniform::u(x[0,1])");
w.factory("mu[100,1,200]");
w.factory("ExtendPdf::p(u,mu)");

w.defineSet("poi","mu");
w.defineSet("obs","x");
//  w.defineSet("obs2","n");
```



Validate on a Poisson

Analytic
RooStats numerical

```
RooJeffreysPrior pi("jeffreys","jeffreys",*w.pdf("p"),*w.set("poi"),*w.set("obs"));
```

Higgs results & look-elsewhere effect

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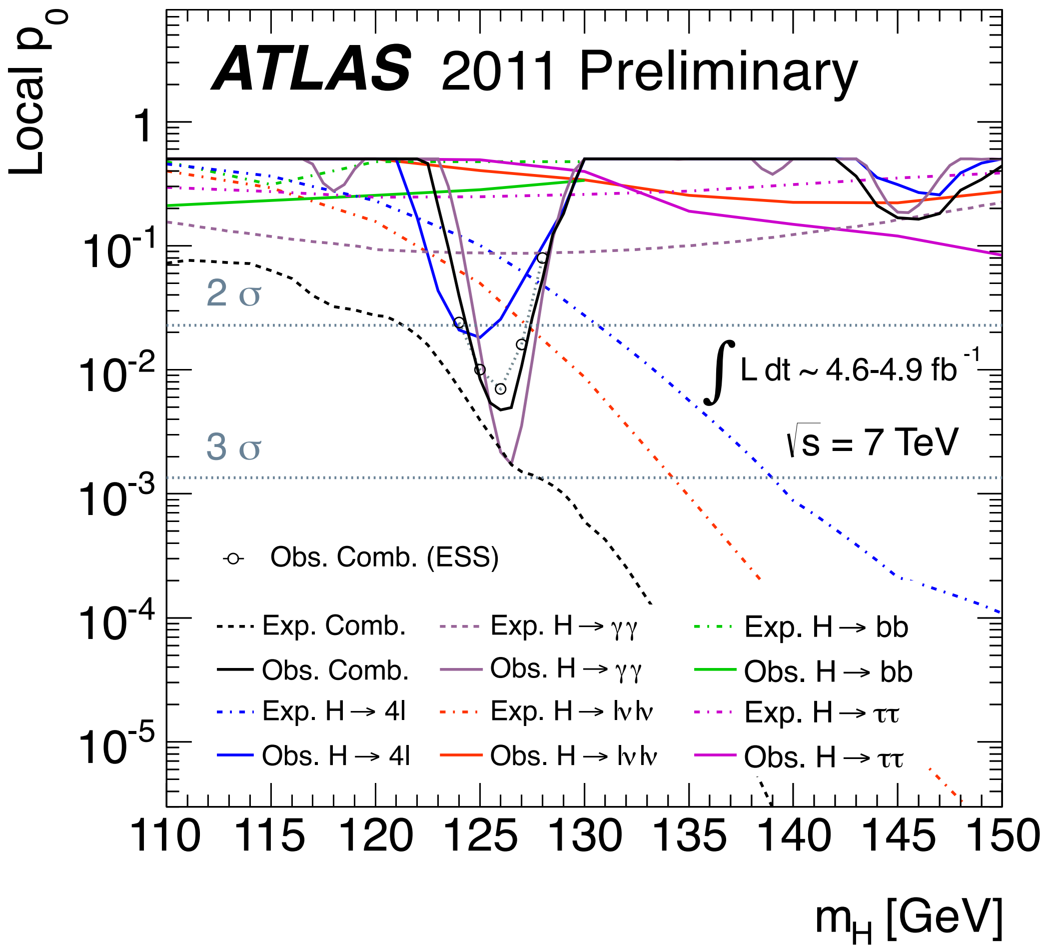
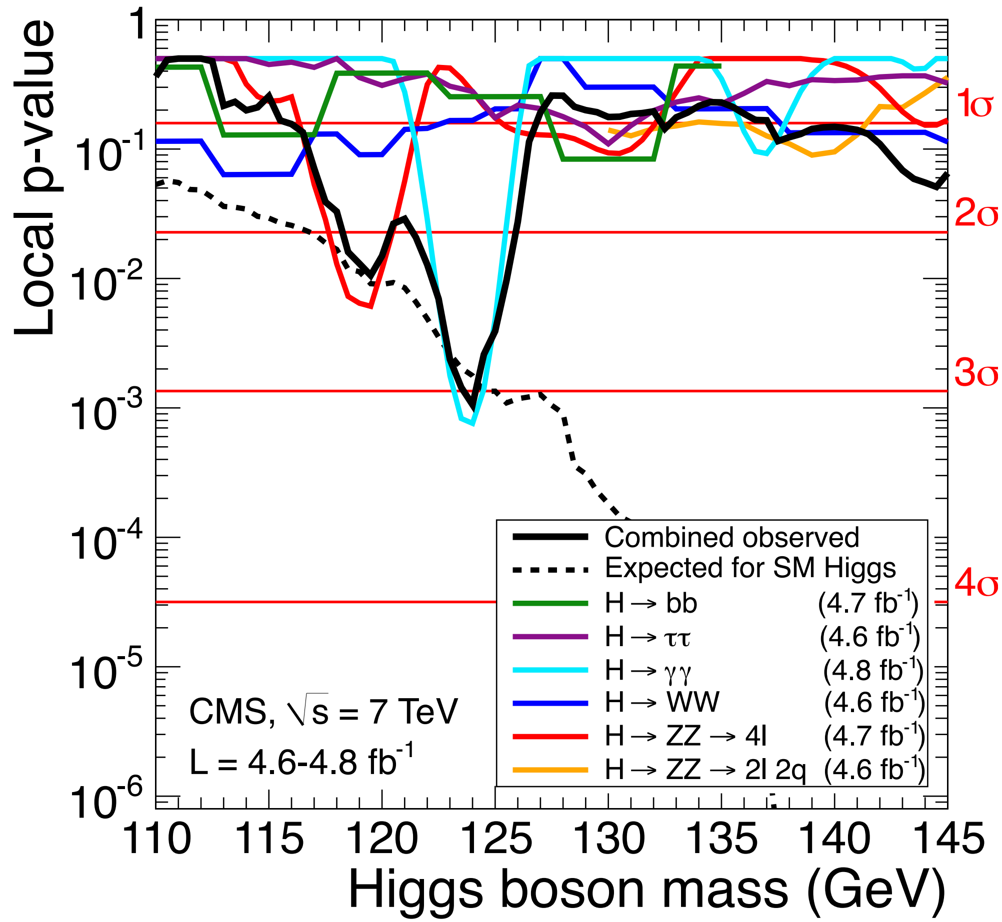
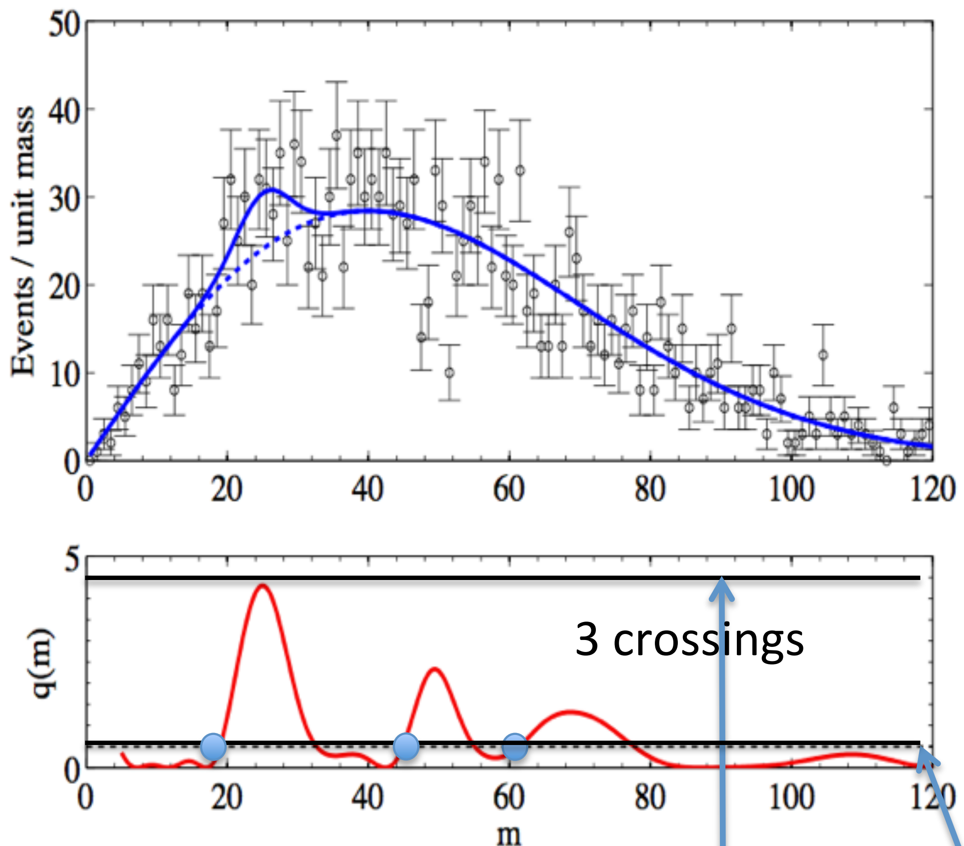
Discovery of the Higgs boson ~ reject null hypothesis.

- Null: background-only   Alternate: signal+background
- Discovery criteria traditionally 5σ ($p < 2.8\ 10^{-7}$)
- Both ATLAS and CMS see ~3σ excess around m_H=125 GeV





Look-elsewhere effect



Typically our signal model has some location parameter, which do not affect the null.

This modifies the distribution of the likelihood ratio test statistic
we call this the "look-elsewhere effect"

Recently Gross & Vitells found the results of Rice, Davies, and Leadbetter for a fast asymptotic approximation for the global p-value

E. Gross & O. Vitells, **Eur.Phys.J. C70 (2010); Astropart.Phys. 35 (2011)**

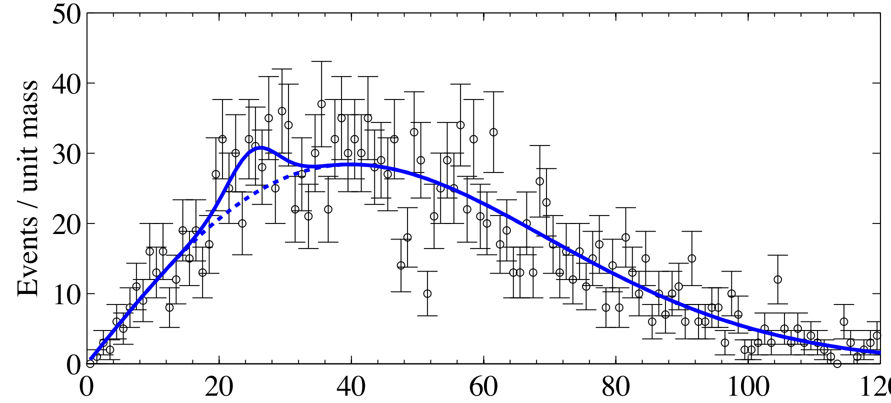
$$p_0^{global} \cong p_0^{local} + \langle N(q_{ref}) \rangle \, e^{-(q_{test}-q_{ref})/2}$$

R. B. Davies,Hypothesis testing when a nuisance parameter is present only under the alternative, Biometrika 64 (1977); Biometrika 74 (1987).

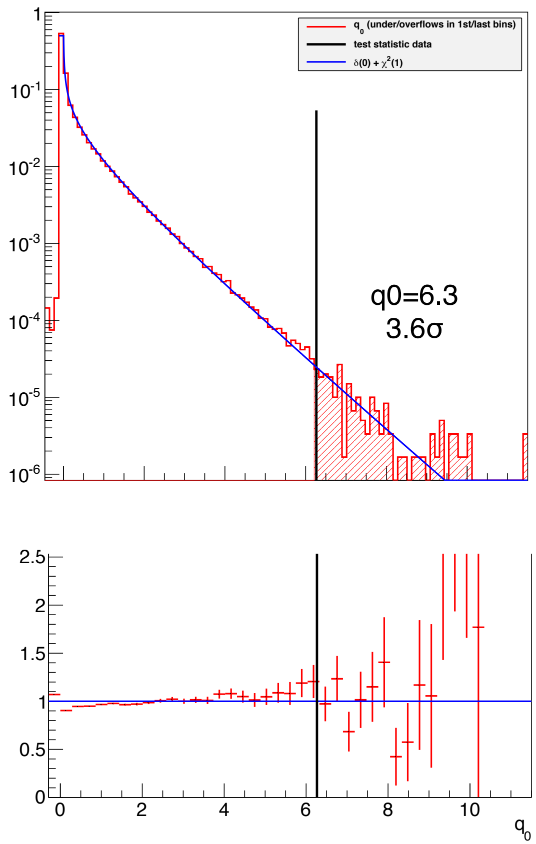
Deviations from the asymptotic distributions

Center for Cosmology and Particle Physics

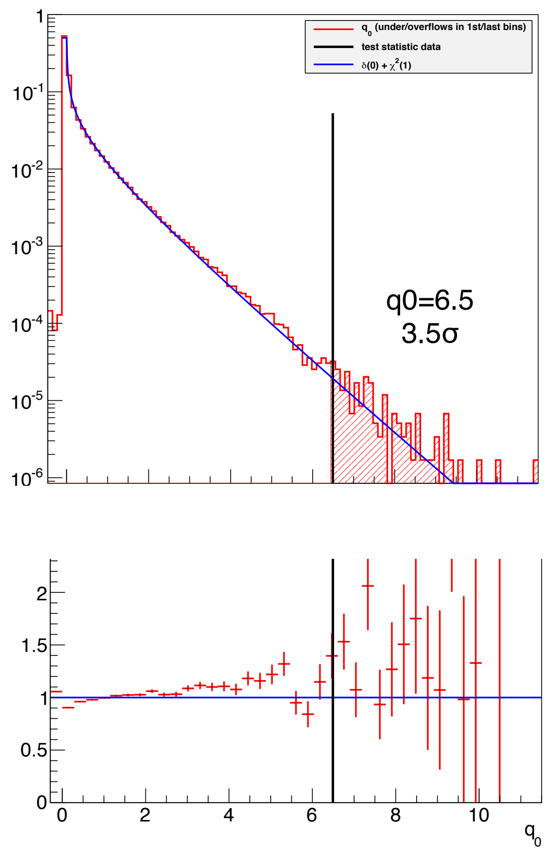
Even if we fix the location of the signal some systematic effects are equivalent to small uncertainty in the location (e.g. energy calibration).



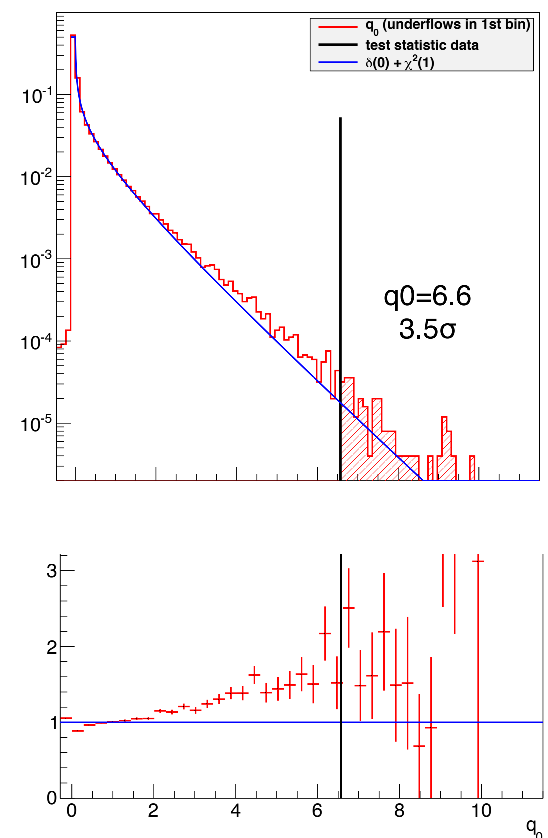
Without energy scale uncertainty
Without mass resolution uncertainty



Without energy scale uncertainty
With mass resolution uncertainty



With energy scale uncertainty
With mass resolution uncertainty



A more subtle effect

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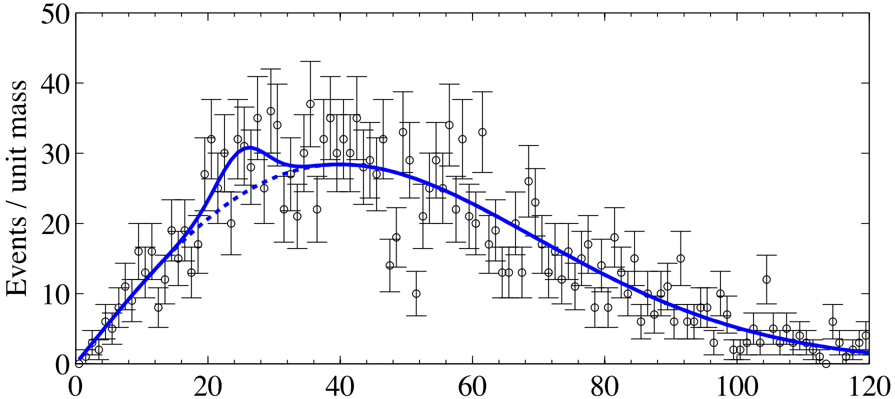
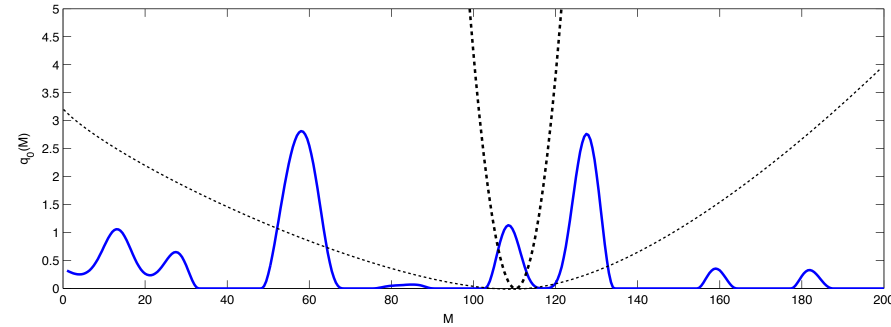


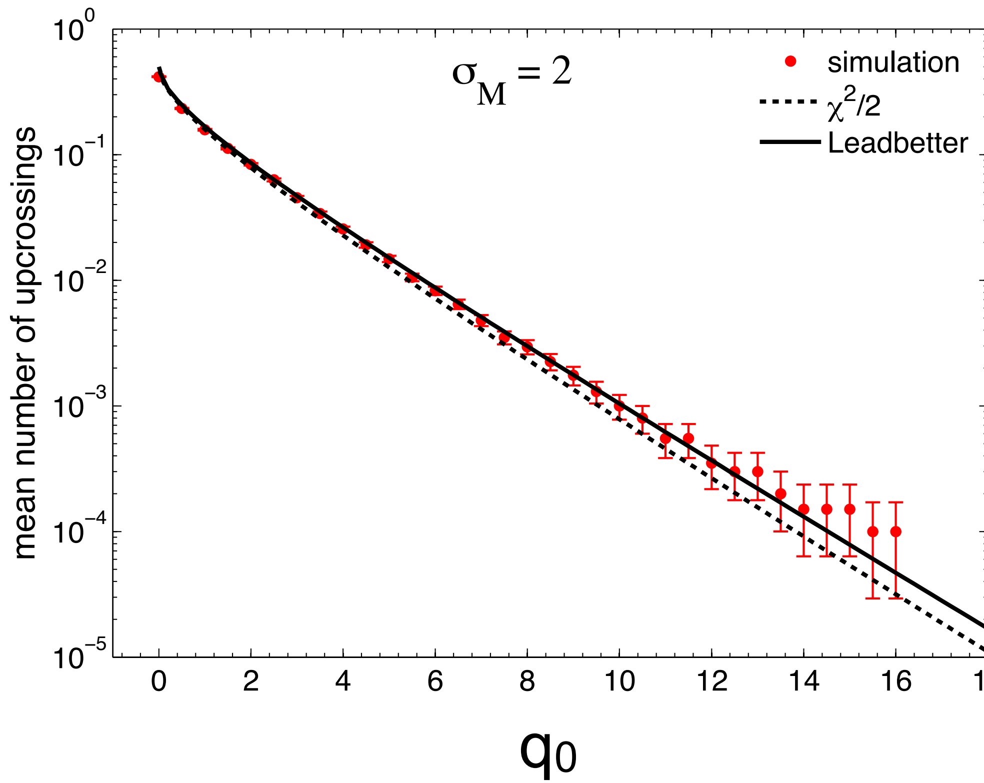
These parameters are slowing convergence to the asymptotic distribution and variance may not reduce with more data.

O. Vitells found exact solution by Leadbetter for the case of only one such nuisance parameter

(H.R. Leadbetter, 1965

$$\mathbb{E}[N_u] = \sigma_2 \int \phi(u(M)) \left[\phi\left(\frac{u'(M)}{\sigma_2}\right) + \frac{u'(M)}{\sigma_2}\left\{\Phi\left(\frac{u'(M)}{\sigma_2}\right) - \frac{1}{2}\right\}\right] dM$$





Review of Challenges
and
Conclusions

Challenges & Difficulties

Interpolation of distributions based on simulated samples with different parameter settings

- currently one-at-a-time variation of nuisance parameters
- need multivariate interpolation procedures
- clearly a weak point of our current modeling approach

As we improve treatment of uncertainties that are statistical in nature, attention turning towards truly systematic uncertainties

- We have something to learn from Brad Efron's talk on about model selection

The complexity of our statistical models is growing exponentially, starting to need new algorithms to deal with them or principles for simplifying them

- graphical models look promising

Publishing likelihood functions is technically possible, but significant sociological and psychological barriers

Conclusions

Particle physics is a unique arena for scientific statistical computing

- relatively classical statistical questions in an extreme setting
- very precise theoretical predictions, lots of data

Collaborative statistical modeling is growing into a powerful scientific tool.

- we are seeing exponential increases in complexity in the models over the last year and a half
- quickly outstripping our tools for maximum likelihood and also our MCMC sampling
- a potential revolution in how we publish our results

Hopefully we will put what we learned from 12 years of PhyStat conferences towards a major discovery soon!

Thank you!